CS 5/7320
Artificial Intelligence

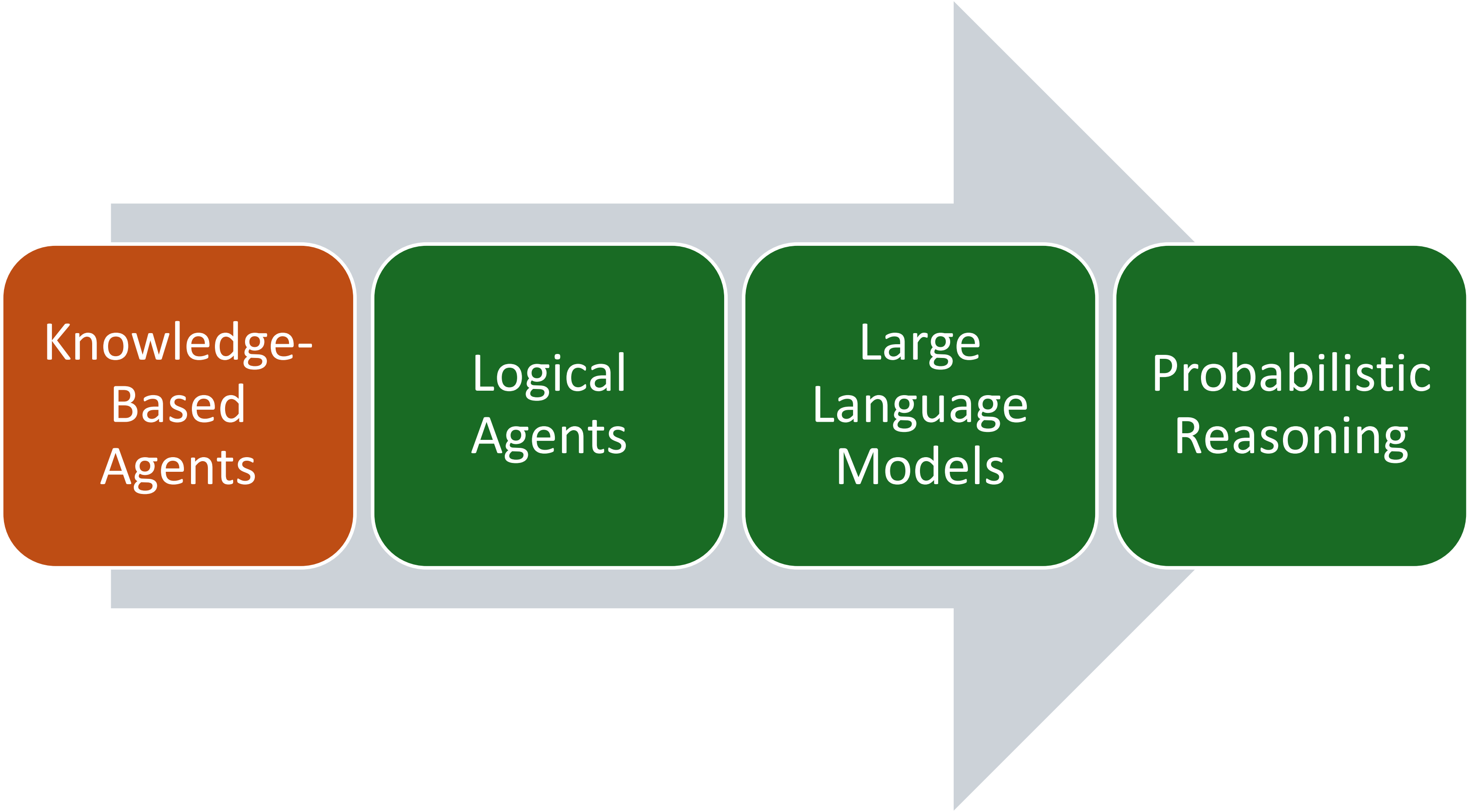
Knowledge-Based Agents
AIMA Chapters 7-9

Slides by Michael Hahsler
based on slides by Svetlana Lazepnik
with figures from the AIMA textbook

"Exercise Plays Vital Role Maintaining Brain Health"
by A Health Blog

Online Material

Outline



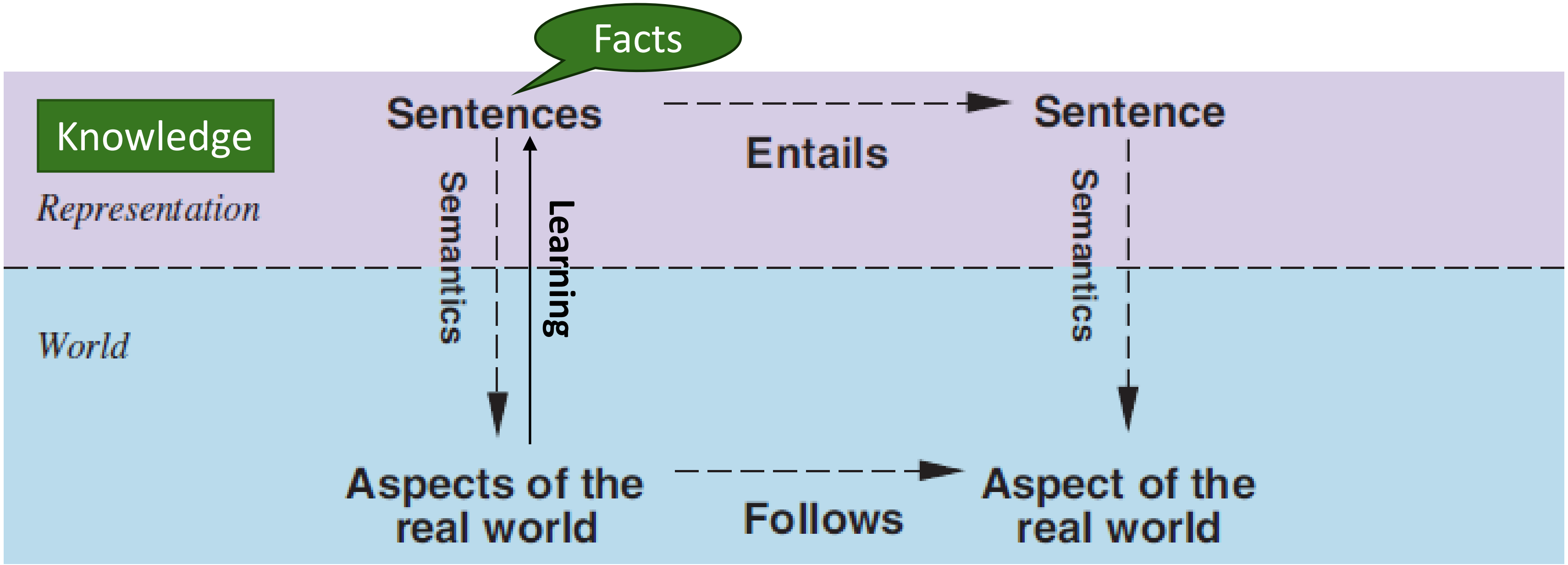
Knowledge-Based Agents

Logical Agents

Large Language Models

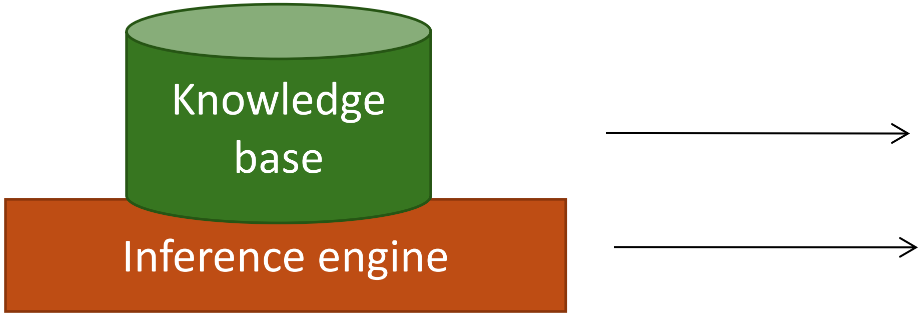
Probabilistic Reasoning

Reality vs. Knowledge Representation



- **Facts:** Sentences we know to be true.
- **Possible worlds**: all worlds/models which are consistent with the facts we know (compare with belief state).
- **Learning** new facts reduces the number of possible worlds.
- **Entailment:** A new sentence logically follows from what we already know.

Knowledge-Based Agents

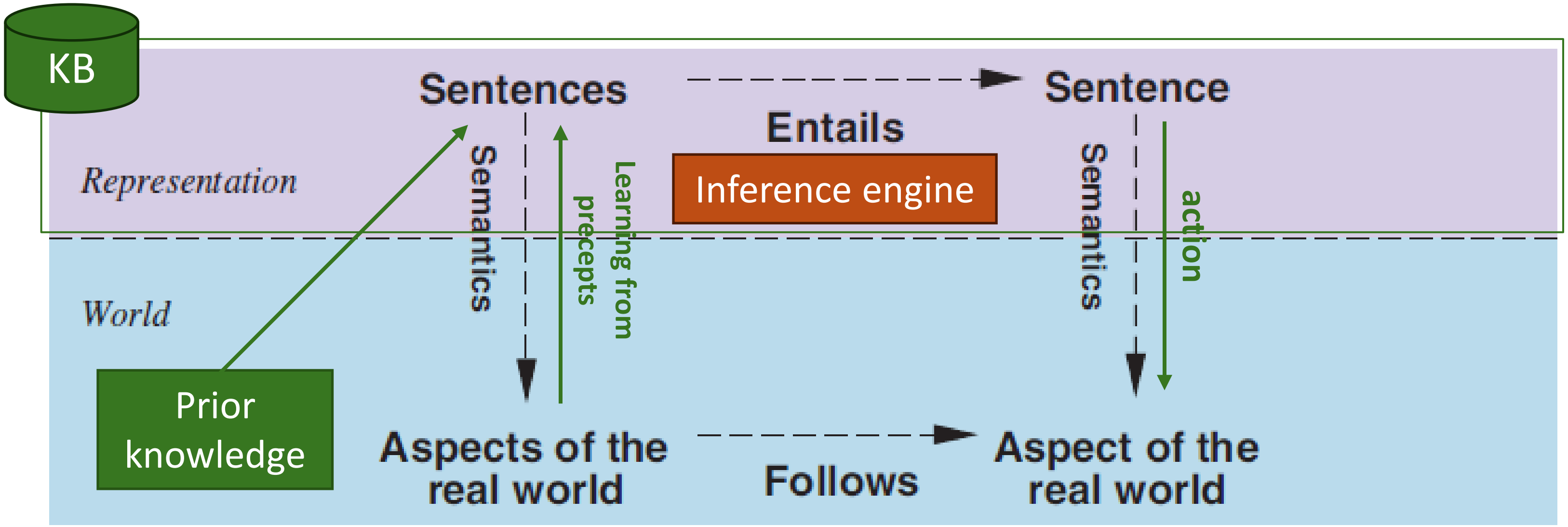


Domain-specific content

Domain-independent algorithms that find new sentences using entailment.

- Knowledge base (KB) = **set of facts.** E.g., set of **sentences** in a **formal language** that are known to be true.
- **Declarative** approach to building an agent: Define what it needs to know in its KB.
- **Separation** between data (knowledge) and program (inference).
- Actions are based on knowledge (sentences + inferred sentences) + an **objective function**. E.g., the agent knows the effects of 5 possible actions and chooses the action with the largest utility.

Generic Knowledge-based Agent



KB

Representation

World

Sentences

Entails

Sentence

Inference engine

Semantics

Learning from precepts

Semantics

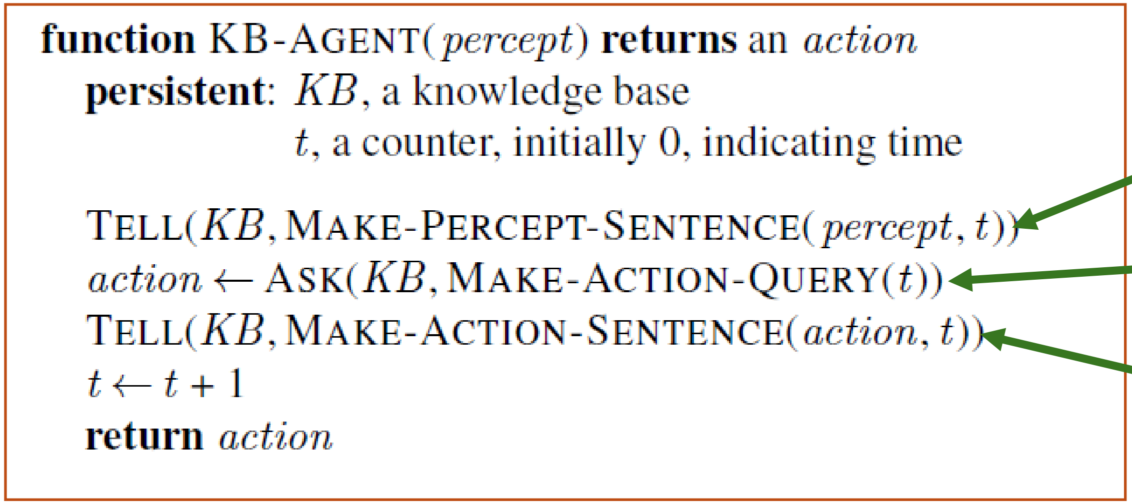
action

Prior knowledge

Aspects of the real world

Follows

Aspect of the real world



```
**function** KB-AGENT(*percept*) **returns** an *action*
   **persistent**: $KB$, a knowledge base
               $t$, a counter, initially 0, indicating time

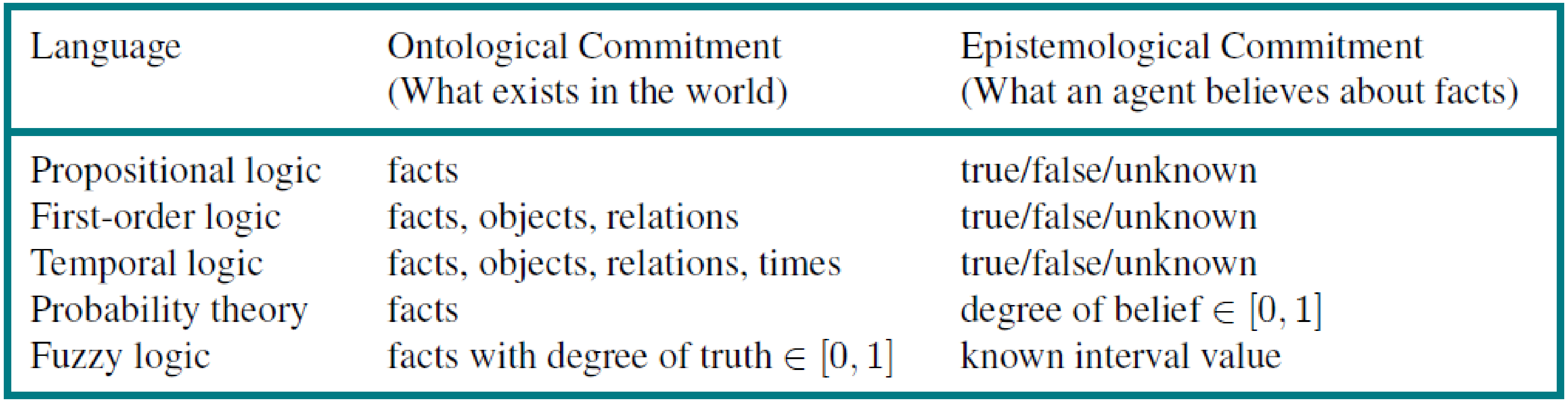
   TELL($KB$, MAKE-PERCEPT-SENTENCE(*percept*, $t$))
   *action* ← ASK($KB$, MAKE-ACTION-QUERY($t$))
   TELL($KB$, MAKE-ACTION-SENTENCE(*action*, $t$))
   $t \leftarrow t + 1$
   **return** *action*
```

Memorize percept at time t

Ask for logical action given an objective

Record action taken at time t

Different Languages to Represent Knowledge



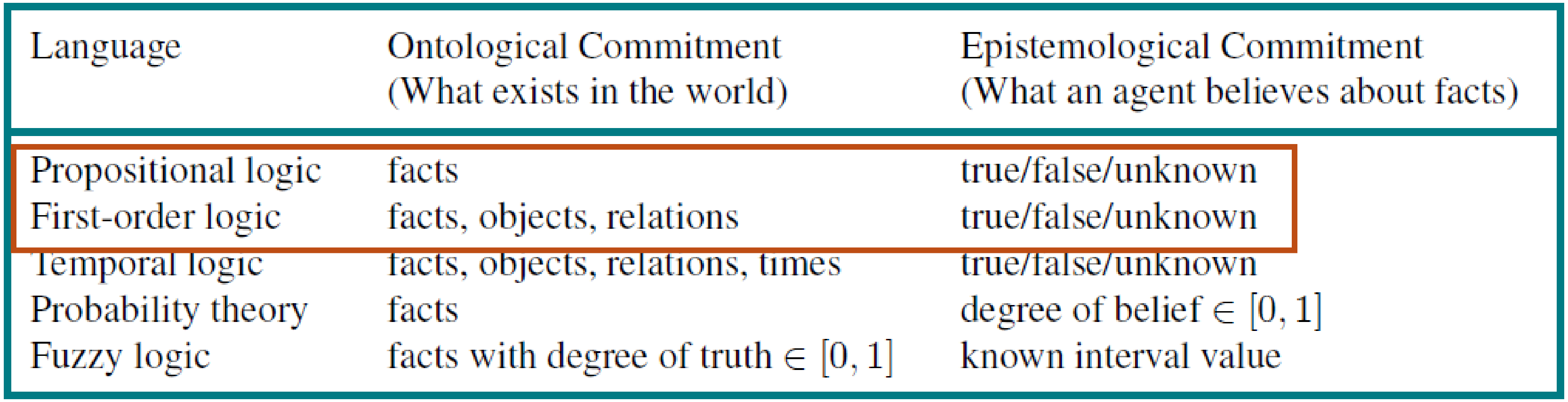
Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

+ Natural Language word patterns representing
facts, objects, relations, … ???

Outline

- Knowledge-Based Agents
- Logical Agents
- Large Language Models
- Probabilistic Reasoning

Logical Agents



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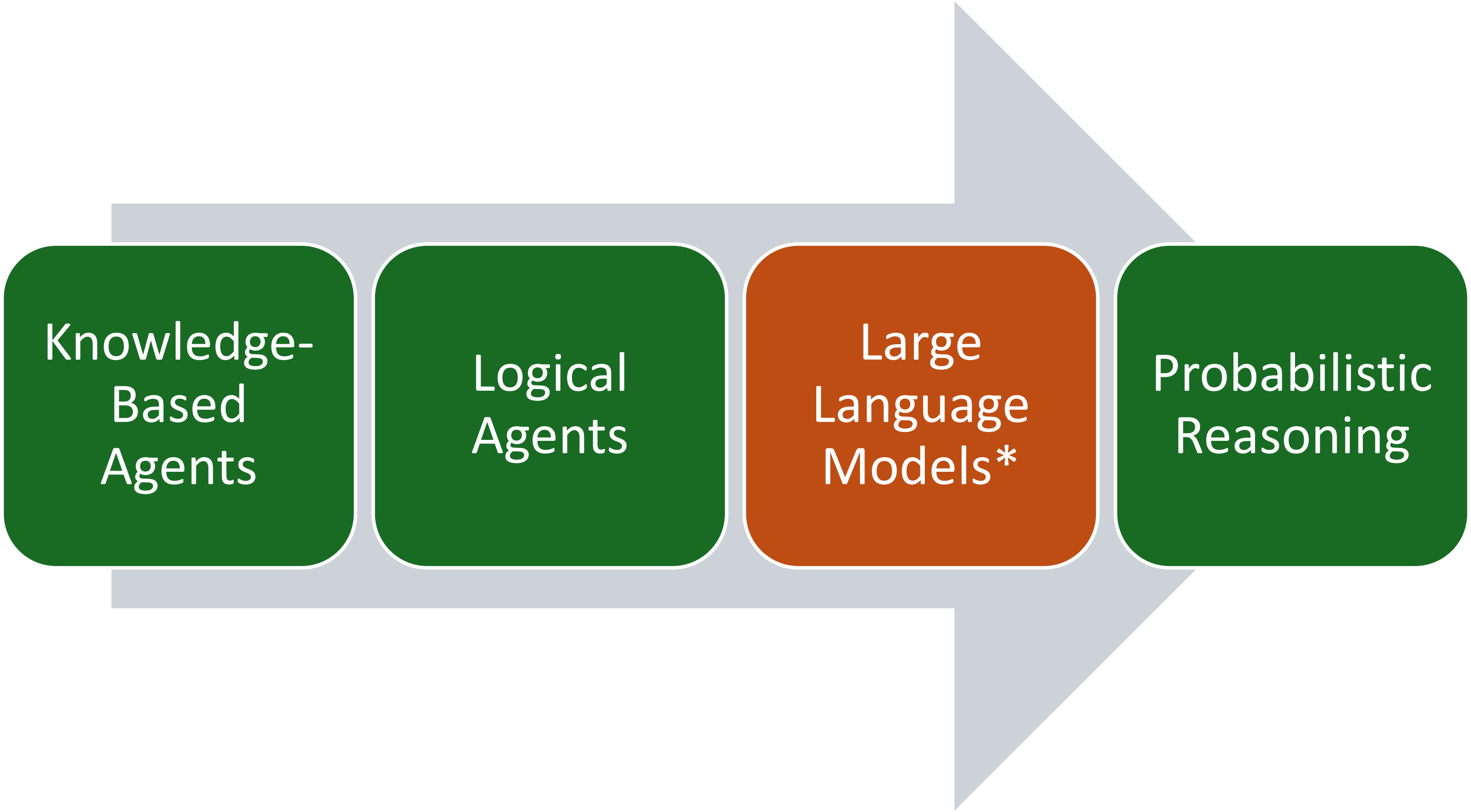
- Facts are logical sentences that are known to be true.
- Inference: Generate new sentences that are entailed by all known sentences.
- Implementation: Typically using Prolog
  - Declarative logic programing language.
  - Runs queries over the program (= the knowledge base)

Issues:

  - Inference is computationally very expensive.
  - Logic cannot deal with uncertainty.

Outline



Knowledge-Based Agents

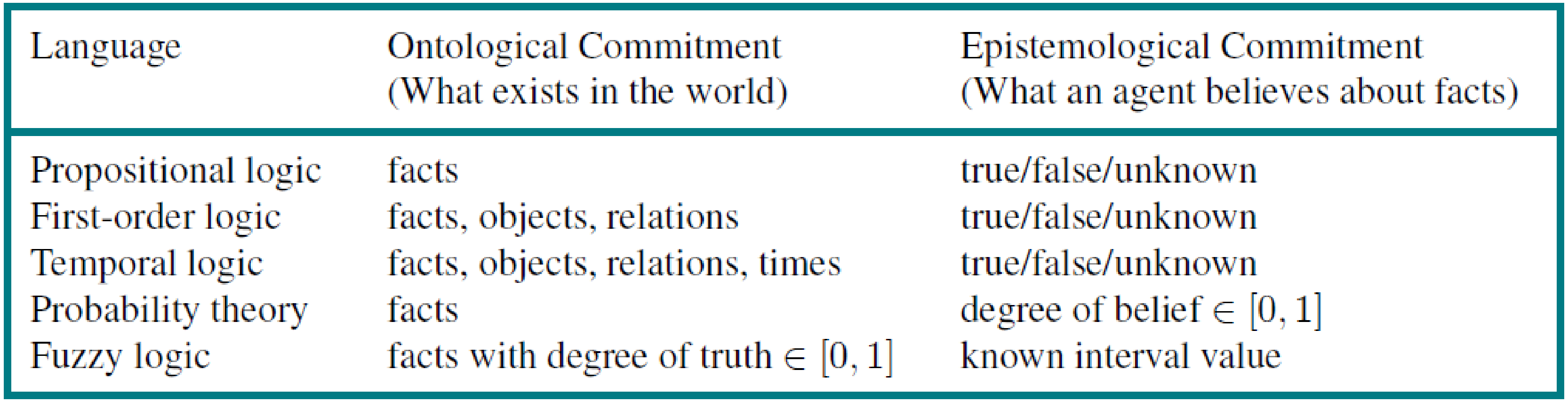
Logical Agents

Large Language Models*

Probabilistic Reasoning

*** This is not in the AIMA textbook!**

LLMs - Large Language Models



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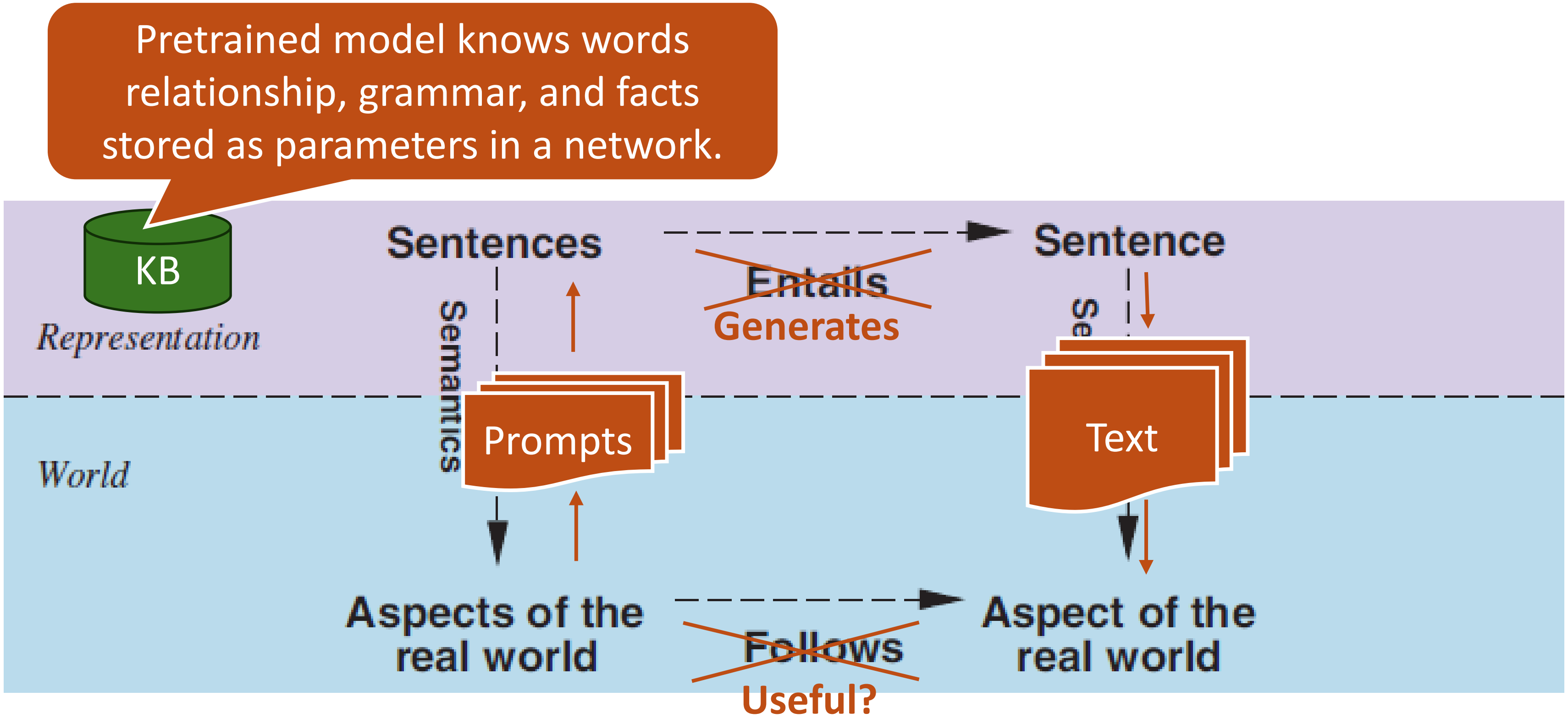
+ Natural Language	word patterns representing facts, objects, relations, …	???

- Store knowledge as parameters in a deep neural networks.

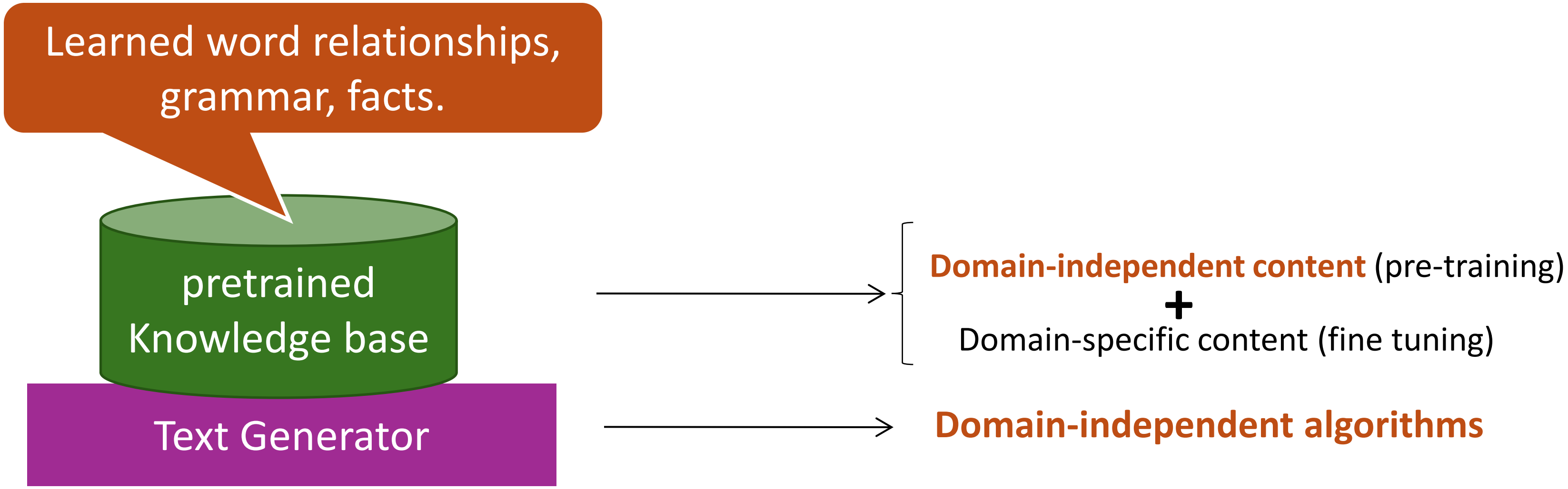
Using Natural Language for Knowledge Representation



Pretrained model knows words relationship, grammar, and facts stored as parameters in a network.

- The user formulates a question about the real world as a natural language prompt (a sequence of tokens).
- The LLM generates text using a model representing its knowledge base.
- The text (hopefully) is useful in the real world. The **objective function** is not clear. Maybe it is implied in the prompt?

LLM as a Knowledge-Based Agents



Domain-independent content (pre-training)
+
Domain-specific content (fine tuning)

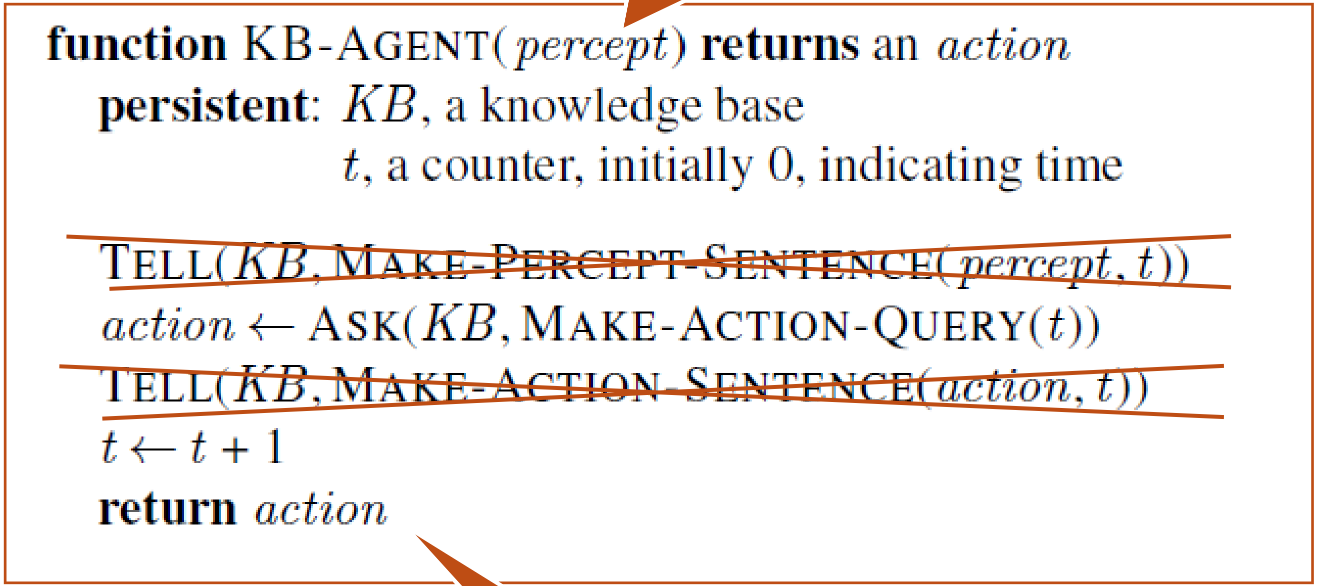
Domain-independent algorithms

Current text generators are:

- Pretrained decoder-only transformer models (e.g., GPT stands for Generative Pre-trained Transformer). The knowledge base is not updated during interactions.
- Tokens are created autoregressively. One token is generated at a time based on all the previous tokens using the transformer attention mechanism.

LLM as a Generic Knowledge-based Agent

Prompt + already generated tokens



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  ~~TELL(KB, MAKE-ACTION-SENTENCE(action, t))~~
  t ← t + 1
  return action
```

Next token

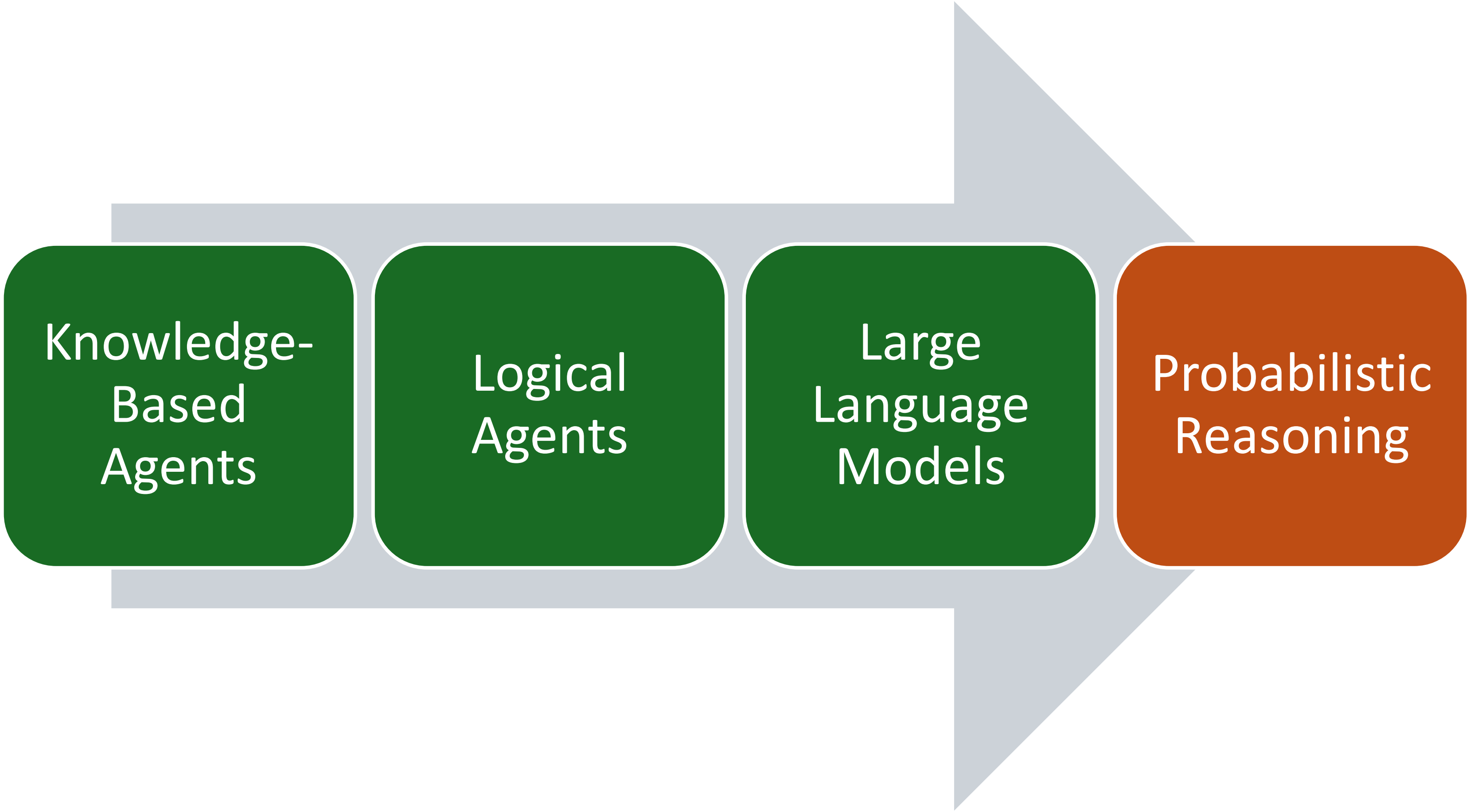
- A chatbot repeatedly calls the agent function till the agent function returns the ‘end’ token.

Many Open Questions about LLMs

- Correlation is not causation: **Can LLMs reason** to solve problems?
- Generative stochasticity leads to **hallucinations**: LLM makes up facts.
- Autoregression is an exponentially **diverging** diffusion process.
- The training data contains **biases**, nonsense and harmful content.
- **Security**: LLM can reveal sensitive information it was trained on.
- **Rights-laundering**: Copyrighted or licensed material can be in the training data.
- Leaky data makes it hard to evaluate true **reasoning performance**.

Reading: [2307.04821] Amplifying Limitations, Harms and Risks of Large Language Models (arxiv.org)

Outline



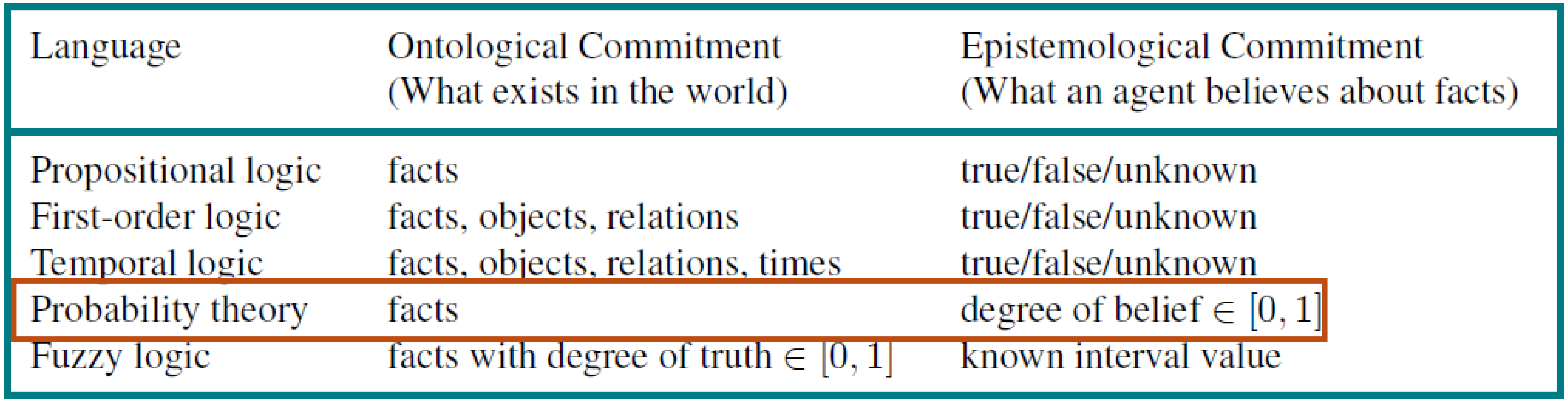
Knowledge-Based Agents

Logical Agents

Large Language Models

Probabilistic Reasoning

Probabilistic Reasoning



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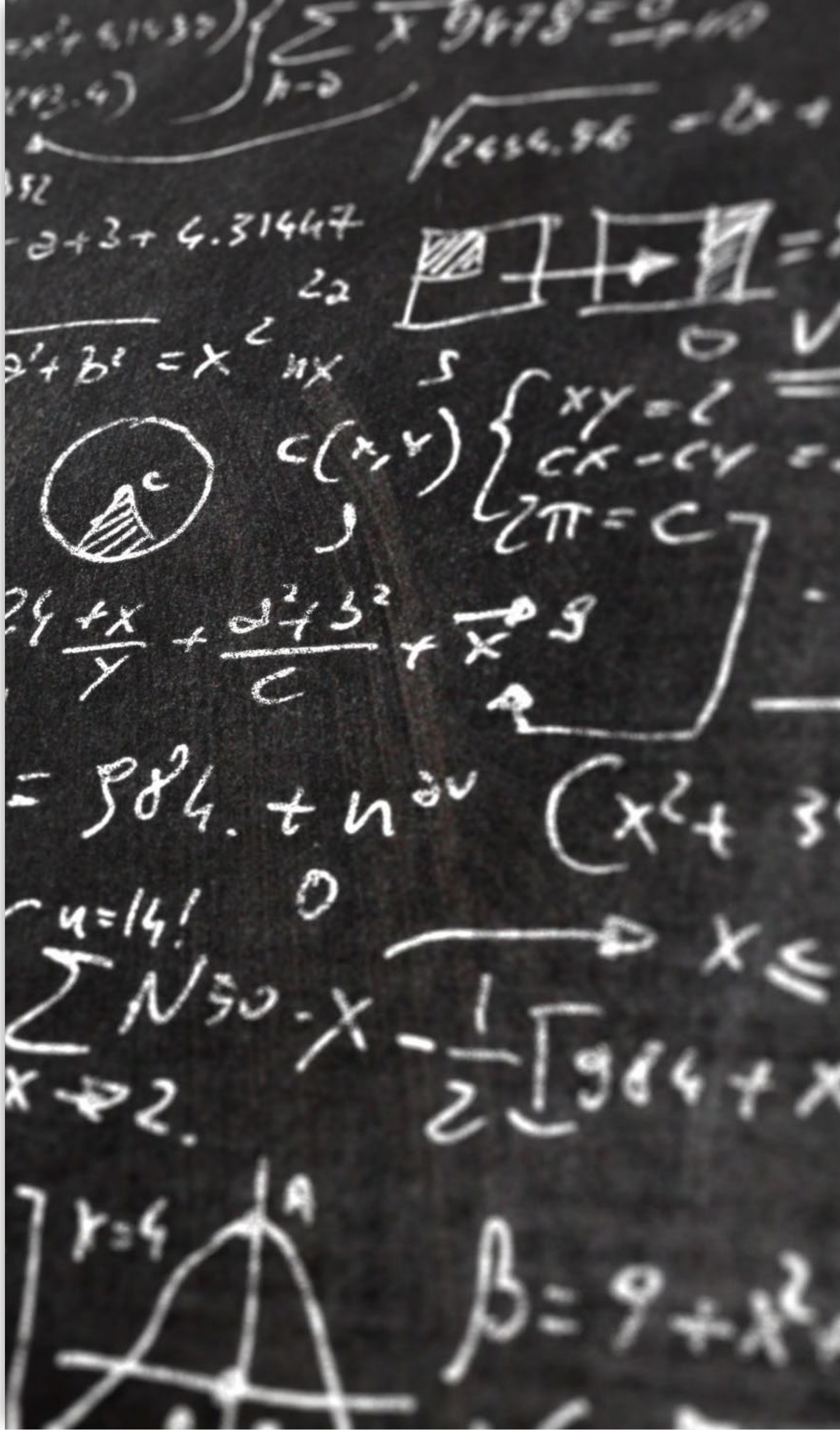
- Replaces true/false with a probability.
- This is the basis for
  - Probabilistic reasoning under uncertainty
  - Decision theory
  - Machine Learning

We will talk about these topics a lot more

Conclusion

- The **clear separation between knowledge and inference engine** is very useful.
- **Pure logic** is often not flexible enough. The fullest realization of knowledge-based agents using logic was in the field of expert systems or knowledge-based systems in the 1970s and 1980s.
- **Pretrained Large Language Models** are an interesting new application of knowledge-based agents based on natural language.
- Next, we will talk about **probability theory** which is the standard language to reason under uncertainty and forms the basis of machine learning.



Appendix: Logic

Details on Propositional and First-Order Logic

Logic to Represent Knowledge

Logic is a formal system for representing and manipulating facts (i.e., knowledge) so that true conclusions may be drawn

Syntax: rules for constructing valid sentences

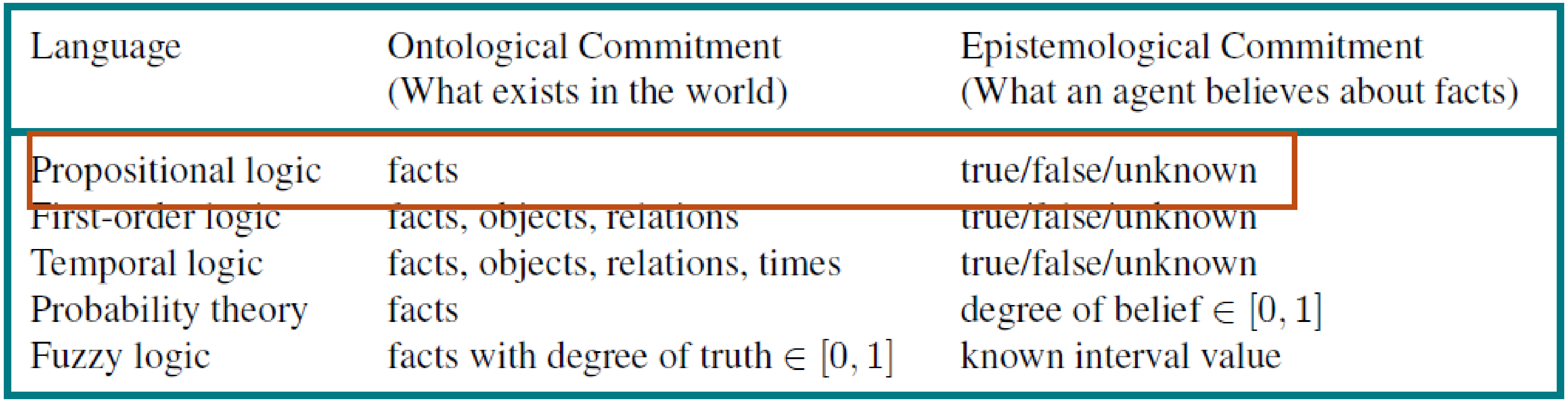
E.g., $x + 2 \geq y$ is a valid arithmetic sentence, $\geq x2y +$ is not

Semantics: "meaning" of sentences, or relationship between logical sentences and the real world

Specifically, semantics defines truth of sentences

E.g., $x + 2 \geq y$ is true in a world where $x = 5$
and $y = 7$

Propositional Logic



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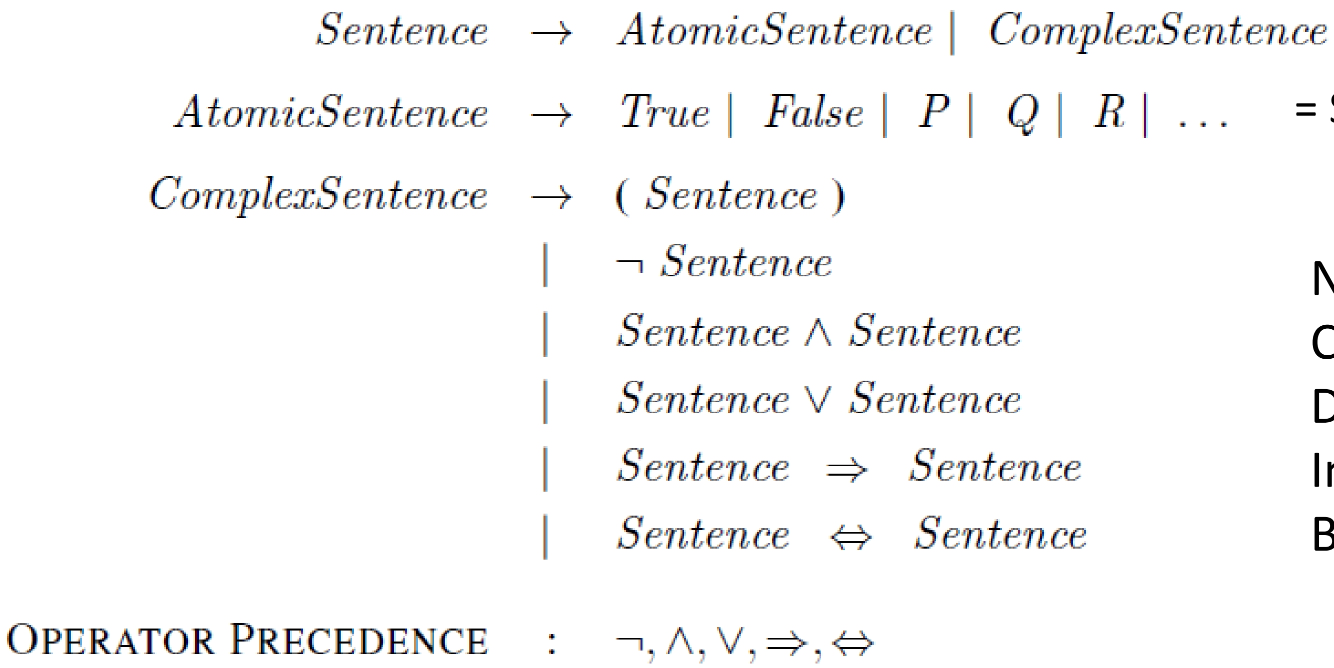
Propositional Logic:
Syntax in Backus-Naur Form



$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots \\
\mathit{ComplexSentence} & \rightarrow & (\ \mathit{Sentence}\ ) \\
 & \mid & \neg\ \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \land \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \lor \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \ \Rightarrow\ \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \ \Leftrightarrow\ \mathit{Sentence} \\
 & & \\
\textsc{Operator Precedence} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

= Symbols

Negation
Conjunction
Disjunction
Implication
Biconditional

Validity and Satisfiability

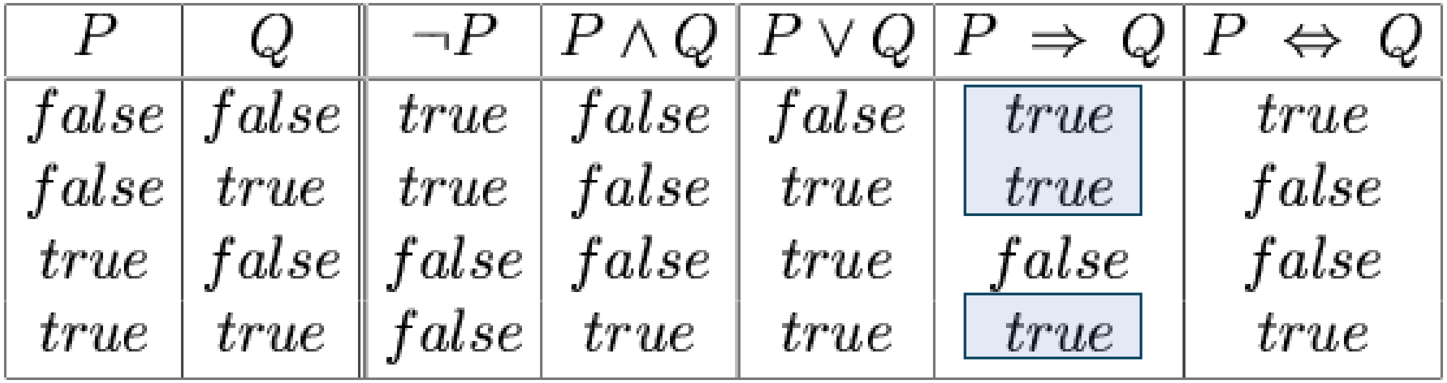
A sentence is **valid** if it is true in **all** models/worlds	e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$ are called tautologies and are useful to deduct new sentences.
A sentence is **satisfiable** if it is true in **some** model	e.g., $A \vee B$, C useful to find new facts that satisfy all current possible worlds.
A sentence is **unsatisfiable** if it is true in no models	e.g., $A \wedge \neg A$

Possible Worlds, Models and Truth Tables

A **model** specifies a “possible world” with the true/false status of each proposition symbol in the knowledge base

- E.g., **P** is true and **Q** is true
- With two symbols, there are $2^2 = 4$ possible worlds/models, and they can be enumerated exhaustively using:

A **truth table** specifies the truth value of a composite sentence for each possible assignments of truth values to its atoms. Each row is a model.



P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

We have 3 possible worlds for $P \Rightarrow Q = true$

Propositional Logic: Semantics

Rules for evaluating truth with respect to a model:

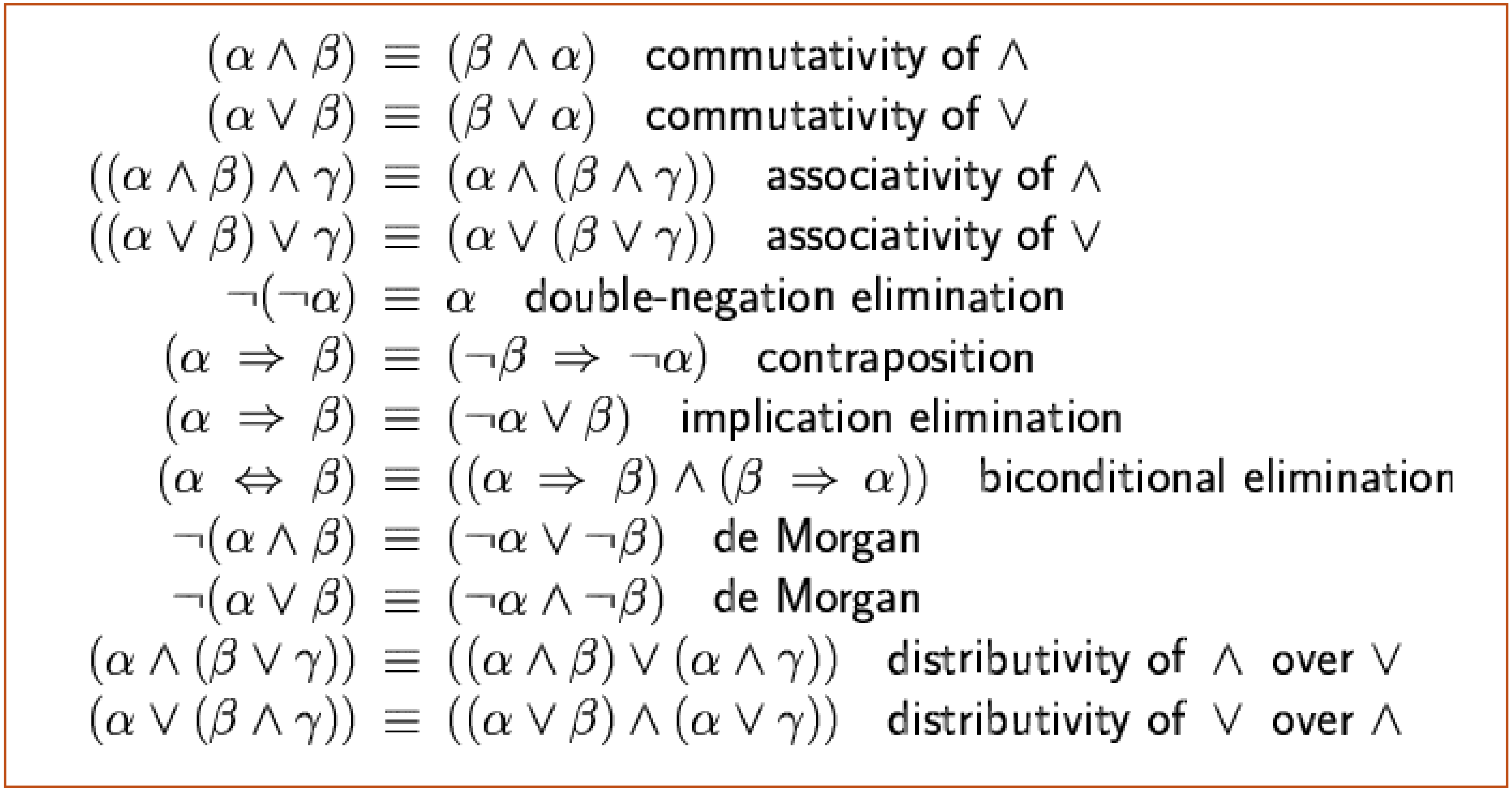
- $\neg \mathbf{P}$ is true iff **P** is false
- $\mathbf{P} \wedge \mathbf{Q}$ is true iff **P** is true and **Q** is true
- $\mathbf{P} \vee \mathbf{Q}$ is true iff **P** is true or **Q** is true
- $\mathbf{P} \Rightarrow \mathbf{Q}$ is true iff **P** is false or **Q** is true

Sentence

Model

Logical Equivalence

Two sentences are logically equivalent iff (read if, and only if) they are true in same models



$(\alpha \land \beta) \equiv (\beta \land \alpha)$ commutativity of $\land$

$(\alpha \lor \beta) \equiv (\beta \lor \alpha)$ commutativity of $\lor$

$((\alpha \land \beta) \land \gamma) \equiv (\alpha \land (\beta \land \gamma))$ associativity of $\land$

$((\alpha \lor \beta) \lor \gamma) \equiv (\alpha \lor (\beta \lor \gamma))$ associativity of $\lor$

$\neg(\neg\alpha) \equiv \alpha$ double-negation elimination

$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$ contraposition

$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \lor \beta)$ implication elimination

$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$ biconditional elimination

$\neg(\alpha \land \beta) \equiv (\neg\alpha \lor \neg\beta)$ de Morgan

$\neg(\alpha \lor \beta) \equiv (\neg\alpha \land \neg\beta)$ de Morgan

$(\alpha \land (\beta \lor \gamma)) \equiv ((\alpha \land \beta) \lor (\alpha \land \gamma))$ distributivity of $\land$ over $\lor$

$(\alpha \lor (\beta \land \gamma)) \equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma))$ distributivity of $\lor$ over $\land$

Entailment

- **Entailment** means that a sentence follows from the premises contained in the knowledge base:

$$KB \models \alpha$$

- The knowledge base *KB* entails sentence α iff α is true in all models where *KB* is true
  - E.g., KB with x = 0 entails sentence x * y = 0
- Tests for entailment
  - $KB \models \alpha$ iff $(KB \Rightarrow \alpha)$ is *valid*
  - $KB \models \alpha$ iff $(KB \wedge \neg\alpha)$ is *unsatisfiable*

Inference

- **Logical inference:** a procedure for generating sentences that follow from (ar entailed by) a knowledge base KB.

- An inference procedure is **sound** if it derives a sentence α iff KB ╞ α. I.e, it only derives **true sentences**.

- An inference procedure is **complete** if it can derive **all** α for which *KB* ╞ α.

Inference

- How can we check whether a sentence α is entailed by KB?
- How about we **enumerate all possible models of the KB** (truth assignments of all its symbols), and check that α is true in every model in which KB is true?
    - This is sound: All produced answer are correct.
    - This is complete: It will produce all correct answers.
    - **Problem**: if KB contains *n* symbols, the truth table will be of size 2^n
- Better idea: use ***inference rules***, or sound procedures to generate new sentences or *conclusions* given the *premises* in the KB.
- Look at the textbook for inference rules and resolution.

Inference Rules

- Modus Ponens

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta} \qquad \begin{matrix} \longleftarrow \quad \text{premises} \\[1em] \longleftarrow \quad \text{conclusion} \end{matrix}$$

This means: If the KB contains the sentences $\alpha \Rightarrow \beta$ and α then β is true.

- And-elimination

$$\frac{\alpha \wedge \beta}{\alpha}$$

Inference Rules

- And-introduction

$$\frac{\alpha,\ \beta}{\alpha \wedge \beta}$$

- Or-introduction

$$\frac{\alpha}{\alpha \vee \beta}$$

Inference Rules

- Double negative elimination

$$\frac{\neg\neg\alpha}{\alpha}$$

- Unit resolution

$$\frac{\alpha \vee \beta, \neg\beta}{\alpha}$$

Resolution

$$\frac{\alpha \vee \beta, \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

or

$$\frac{\alpha \vee \beta, \beta \Rightarrow \gamma}{\alpha \vee \gamma}$$

- Example:
  - α: “The weather is dry”
  - β: “The weather is rainy”
  - γ: “I carry an umbrella”

Resolution is Complete

$$\frac{\alpha \vee \beta, \neg \beta \vee \gamma}{\alpha \vee \gamma}$$

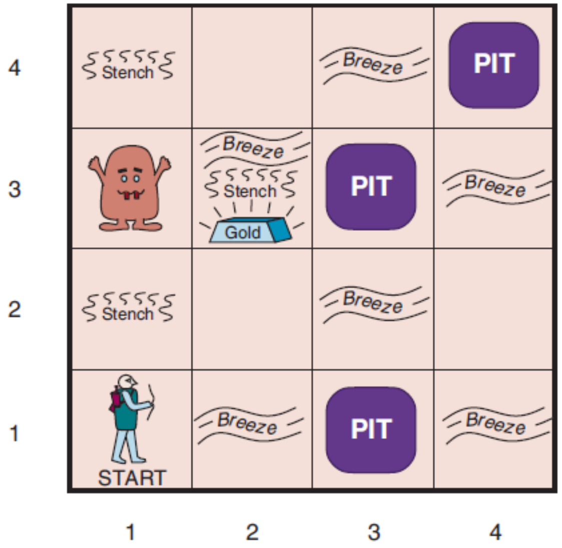
- To prove $KB \models \alpha$, assume $KB \wedge \neg \alpha$ and derive a contradiction
- Rewrite $KB \wedge \neg \alpha$ as a conjunction of *clauses*, or disjunctions of *literals*
  - *Conjunctive normal form* (CNF)
- Keep applying resolution to clauses that contain *complementary literals* and adding resulting clauses to the list
  - If there are no new clauses to be added, then KB does not entail α
  - If two clauses resolve to form an *empty clause*, we have a contradiction and $KB \models \alpha$

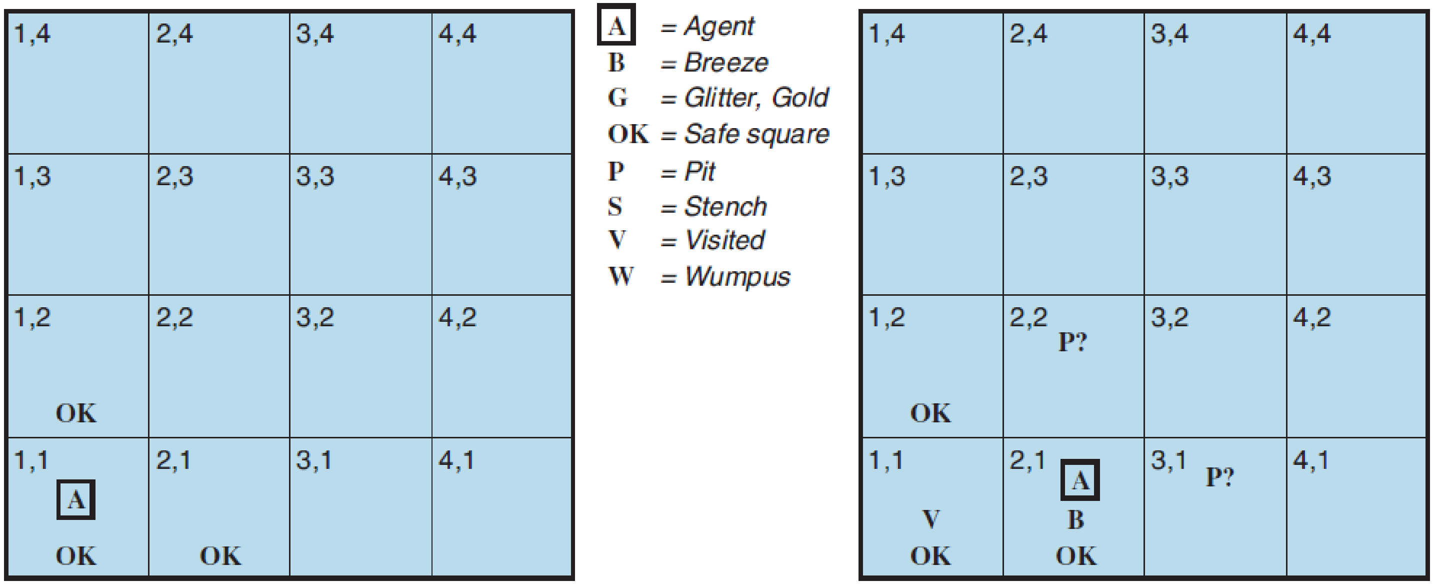
Complexity of Inference

- Propositional inference is ***co-NP-complete***
  - *Complement* of the SAT problem: $\alpha \models \beta$ if and only if the sentence $\alpha \land \neg \beta$ is *unsatisfiable*
  - Every known inference algorithm has worst-case exponential run time complexity.

- Efficient inference is only possible for restricted cases
  - e.g., Horn clauses are disjunctions of literals with at most one positive literal.

Example: Wumpus World





(a)

(b)

Example: Wumpus World

Initial KB needs to contain rules like these for each square:

$Breeze(1,1) \Leftrightarrow Pit(1,2) \lor Pit(2,1)$
$Breeze(1,2) \Leftrightarrow Pit(1,1) \lor Pit(1,3) \lor Pit(2,2)$
$\text{Stench}(1,1) \Leftrightarrow W(1,2) \lor W(2,1)$
…

We have to enumerate all possible scenarios in propositional logic! First-order logic can help.

Percepts at (1,1) are no breeze or stench. Add the following facts to the KB:

$\neg Breeze(1,1)$
$\neg Stench(1,1)$

Inference will tell us that the following facts are entailed:

$\neg Pit(1,2), \neg Pit(2,1), \neg W(1,2), \neg W(2,1)$

This means that (1,2) and (2,1) are safe.

Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions.
- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences in models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
- Resolution is complete for propositional logic.
- Algorithms use forward, backward chaining, are linear in time, and complete for special clauses (definite clauses).

Limitations of Propositional Logic

Suppose you want to say “All humans are mortal”

- In propositional logic, you would need ~6.7 billion statements of the form:

  Michael_Is_Human and Michael_Is_Mortal,

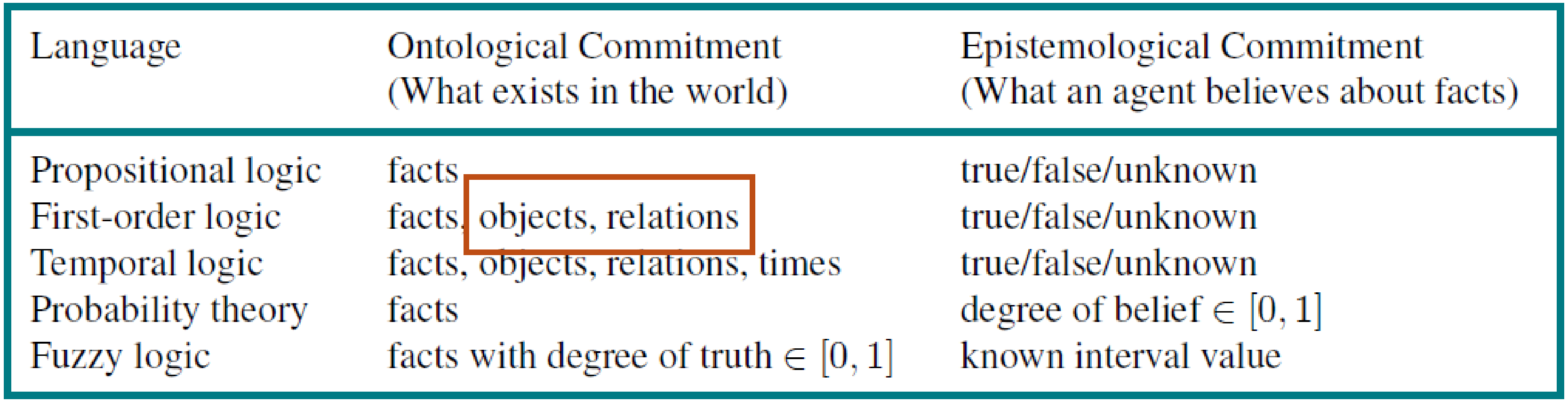
  Sarah_Is_Human and Sarah_Is_Mortal, …

Suppose you want to say “Some people can run a marathon”

- You would need a disjunction of ~6.7 billion statements:

  Michael_Can_Run_A_Marathon or … or Sarah_Can_Run_A_Marathon

First-Order Logic



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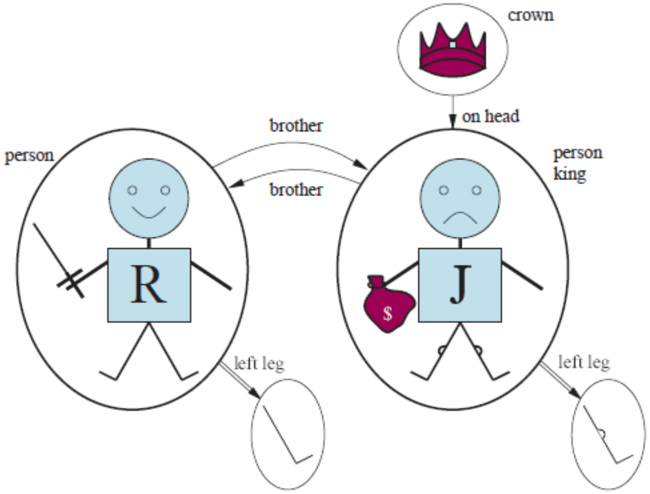
First-order Logic adds **objects** and **relations** to the facts of propositional logic.

This addresses the issues of propositional logic, which needs to store a fact for each instance of and object individually.

Syntax of FOL



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 & \mid & \textit{Sentence} \ \Leftrightarrow\ \textit{Sentence} \\
 & \mid & \textit{Quantifier}\ \textit{Variable}, \ldots\ \textit{Sentence} \\
 & & \\
\textit{Term} & \rightarrow & \textit{Function}(\textit{Term}, \ldots) \\
 & \mid & \textit{Constant} \\
 & \mid & \textit{Variable} \\
 & & \\
\textit{Quantifier} & \rightarrow & \forall \mid \exists \\
\textit{Constant} & \rightarrow & A \mid X_1 \mid \textit{John} \mid \cdots \\
\textit{Variable} & \rightarrow & a \mid x \mid s \mid \cdots \\
\textit{Predicate} & \rightarrow & \textit{True} \mid \textit{False} \mid \textit{After} \mid \textit{Loves} \mid \textit{Raining} \mid \cdots \\
\textit{Function} & \rightarrow & \textit{Mother} \mid \textit{LeftLeg} \mid \cdots \\
\textrm{OPERATOR PRECEDENCE} & : & \neg, =, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

Objects

Relations. Predicate is/returns True or False

Function returns an object

Universal Quantification

- **$\forall$x P(x)**
- Example: “Everyone at SMU is smart”
  $\forall$x At(x,SMU) $\Rightarrow$ Smart(x)
  Why not **$\forall$x At(x,SMU) $\wedge$ Smart(x)**?
- Roughly speaking, equivalent to the conjunction of all possible instantiations of the variable:
  [At(John, SMU) $\Rightarrow$ Smart(John)] $\wedge$ ...
  [At(Richard, SMU) $\Rightarrow$ Smart(Richard)] $\wedge$ ...
- **$\forall$x P(x)** is true in a model m iff **P(x)** is true with **x** being each possible object in the model

Existential Quantification

- **∃x P(x)**
- Example: “Someone at SMU is smart”

  ∃x At(x,SMU) ∧ Smart(x)

  Why not **∃x At(x,SMU) ⇒ Smart(x)**?
- Roughly speaking, equivalent to the disjunction of all possible instantiations:

  [At(John,SMU) ∧ Smart(John)] ∨

  [At(Richard,SMU) ∧ Smart(Richard)] ∨ …
- **∃x P(x)** is true in a model m iff **P(x)** is true with **x** being some possible object in the model

Properties of Quantifiers

- **∀x ∀y** is the same as **∀y ∀x**
- **∃x ∃y** is the same as **∃y ∃x**
- **∃x ∀y** is not the same as **∀y ∃x**
  - **∃x ∀y Loves(x,y)**
    - “There is a person who loves everyone”
  - **∀y ∃x Loves(x,y)**
    - “Everyone is loved by at least one person”

- **Quantifier duality:** each quantifier can be expressed using the other with the help of negation
  - **∀x Likes(x,IceCream)** **¬∃x**
  - **∃x Likes(x,Broccoli)** **¬∀**

Equality

- **$Term_1$ = $Term_2$** is true under a given model if and only if **$Term_1$** and **$Term_2$** refer to the same object

- E.g., definition of **Sibling** in terms of **Parent**:

  $\forall$**x,y Sibling(x,y)** $\Leftrightarrow$
  [$\neg$**(x = y)** $\wedge$ $\exists$**m,f** $\neg$ **(m = f)** $\wedge$ **Parent(m,x)** $\wedge$ **Parent(f,x)** $\wedge$
  Parent(m,y) $\wedge$ **Parent(f,y)]**

Example: The Kinship Domain

- Brothers are siblings

  $\forall$**x,y Brother(x,y)** $\Rightarrow$ **Sibling(x,y)**

- "Sibling" is symmetric

  $\forall$**x,y Sibling(x,y)** $\Leftrightarrow$ **Sibling(y,x)**

- One's mother is one's female parent

  $\forall$**m,c (Mother(c) = m)** $\Leftrightarrow$ **(Female(m)** $\wedge$ **Parent(m,c))**

Example: The Set Domain

- **$\forall s\ Set(s) \Leftrightarrow (s = \{\}) \lor (\exists x, s_2\ Set(s_2) \land s = \{x|s_2\})$**
- **$\neg\exists x, s\ \{x|s\} = \{\}$**
- **$\forall x, s\ x \in s \Leftrightarrow s = \{x|s\}$**
- **$\forall x, s\ x \in s \Leftrightarrow [\ \exists y, s_2\ (s = \{y|s_2\} \land (x = y \lor x \in s_2))]$**
- **$\forall s_1, s_2\ s_1 \subseteq s_2 \Leftrightarrow (\forall x\ x \in s_1 \Rightarrow x \in s_2)$**
- **$\forall s_1, s_2\ (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \land s_2 \subseteq s_1)$**
- **$\forall x, s_1, s_2\ x \in (s_1 \cap s_2) \Leftrightarrow (x \in s_1 \land x \in s_2)$**
- **$\forall x, s_1, s_2\ x \in (s_1 \cup s_2) \Leftrightarrow (x \in s_1 \lor x \in s_2)$**

Inference in FOL

Inference in FOL is complicated!

1. **Reduction to propositional logic** and then use propositional logic inference.

2. **Directly do inference on FOL (or a subset like definite clauses)**
    - Unification: Combine two sentences into one.
    - Forward Chaining for FOL
    - Backward Chaining for FOL
    - Logical programming (e.g., Prolog)