CS 5/7320 Artificial Intelligence

Knowledge-Based Agents
AIMA Chapters 7-9

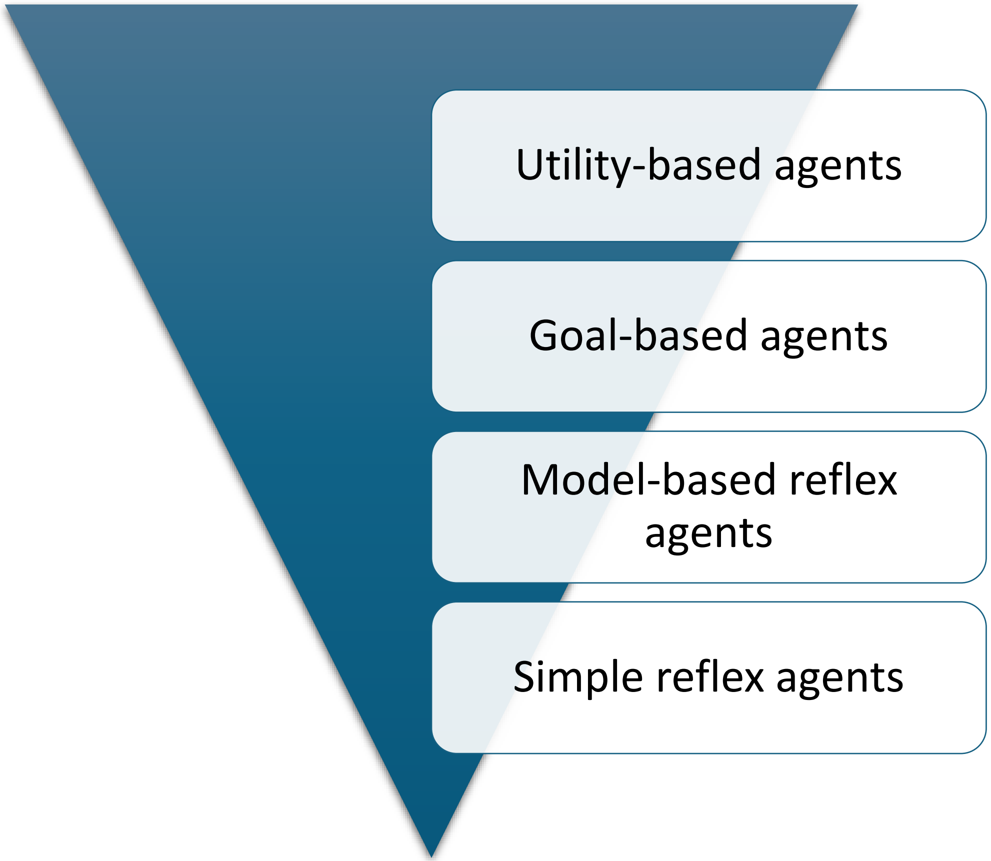
Slides by Michael Hahsler

based on slides by Svetlana Lazepnik
with figures from the AIMA textbook

Image: "Exercise Plays Vital Role Maintaining Brain Health" by A Health Blog

Online Material

Agent Types So Far

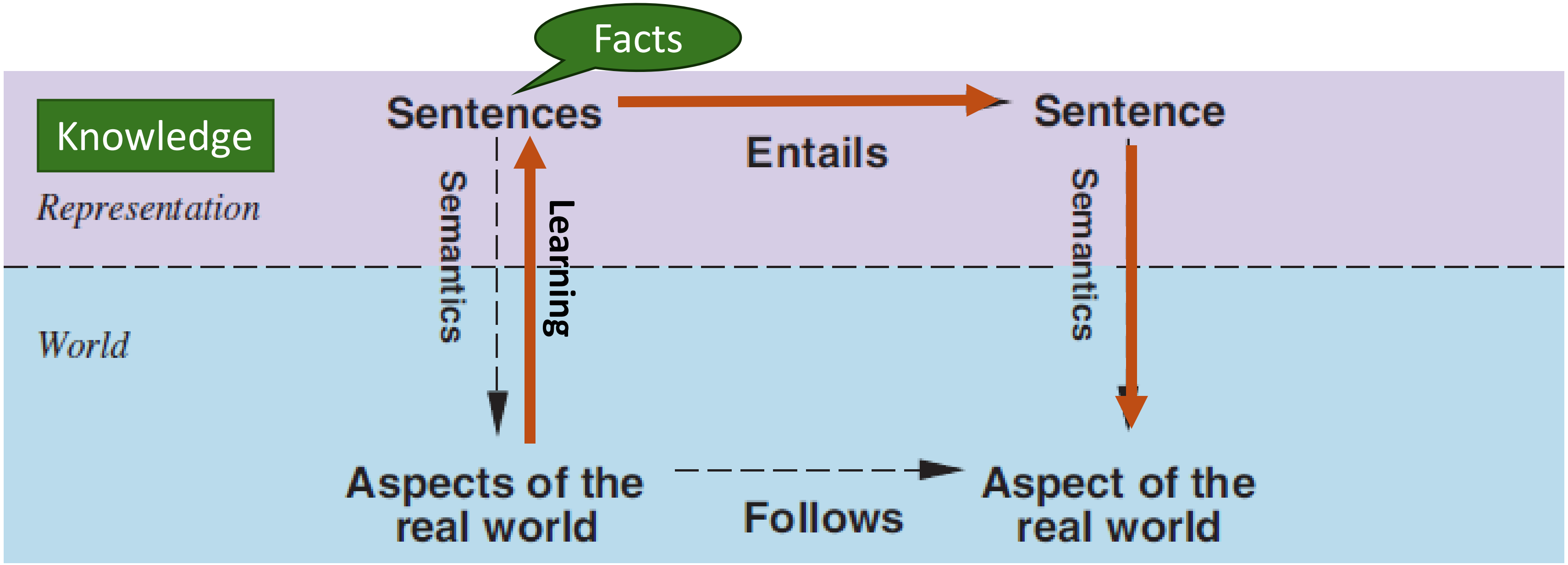


We will introduce a different class of agents that relies on reasoning using knowledge.

Outline

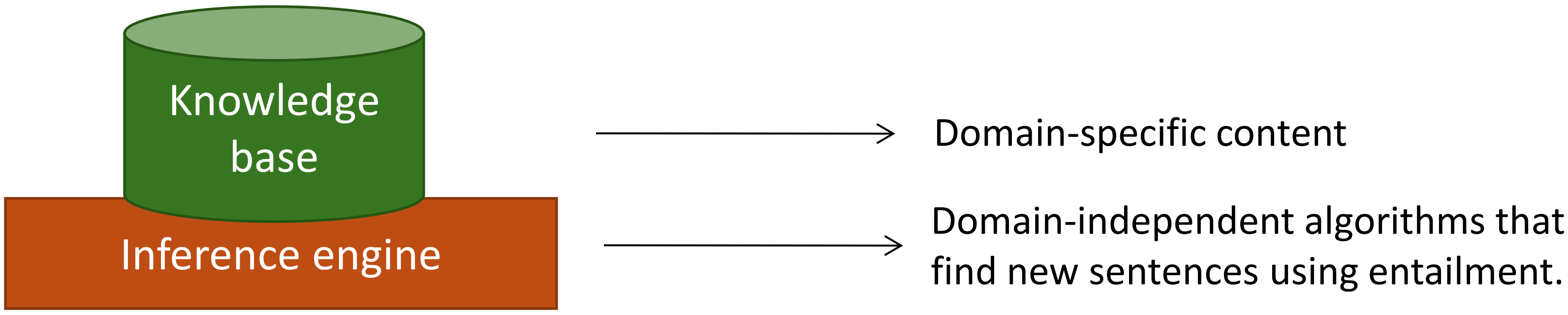
- Knowledge-Based Agents
- Logical Agents
- Probabilistic Reasoning Agents
- Large Language Models

Reality vs. Knowledge Representation



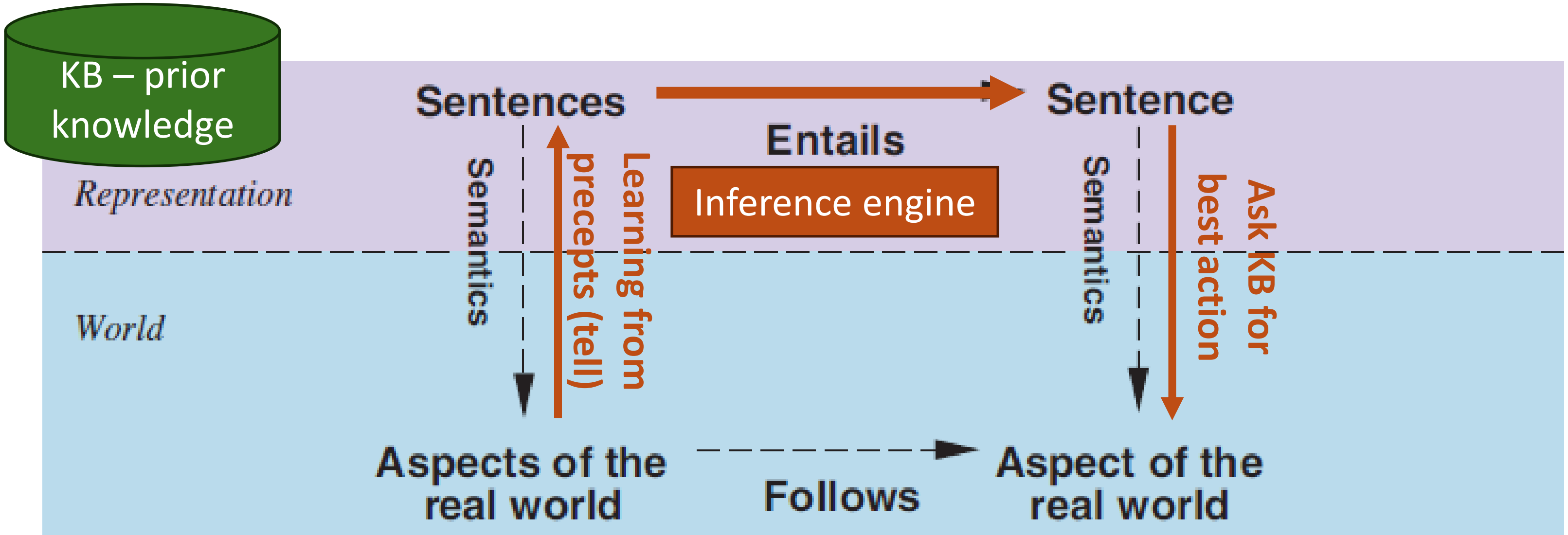
- **Facts:** Sentences we know to be true.
- **Possible worlds**: all worlds/models which are consistent with the facts we know (compare with belief state).
- **Learning** new facts reduces the number of possible worlds.
- **Entailment:** A new sentence logically follows from what we already know.
- **Reasoning**: The agent can reason about what will happen in the real world.

Knowledge-Based Agents



- Knowledge base (KB) = **set of facts.** E.g., set of **sentences** in a **formal language** that are known to be true.
- **Separation** between data (knowledge) and program (inference).
- **Declarative** approach to building an agent: Define what it needs to know in its KB. Use an off-the-shelf inference engine.
- The concepts of **goals or utility** can be incorporated into the knowledge base as facts.
- **Actions** are based on knowledge (sentences + inferred sentences).

Generic Knowledge-based Agent




```
function KB-AGENT(percept) returns an action
  persistent: KB, a knowledge base
              t, a counter, initially 0, indicating time

  TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
  action ← ASK(KB, MAKE-ACTION-QUERY(t))
  TELL(KB, MAKE-ACTION-SENTENCE(action, t))
  t ← t + 1
  return action
```

Memorize percept at time t

Ask for logical action given the knowledge

Record action taken at time t

Different Languages to Represent Knowledge

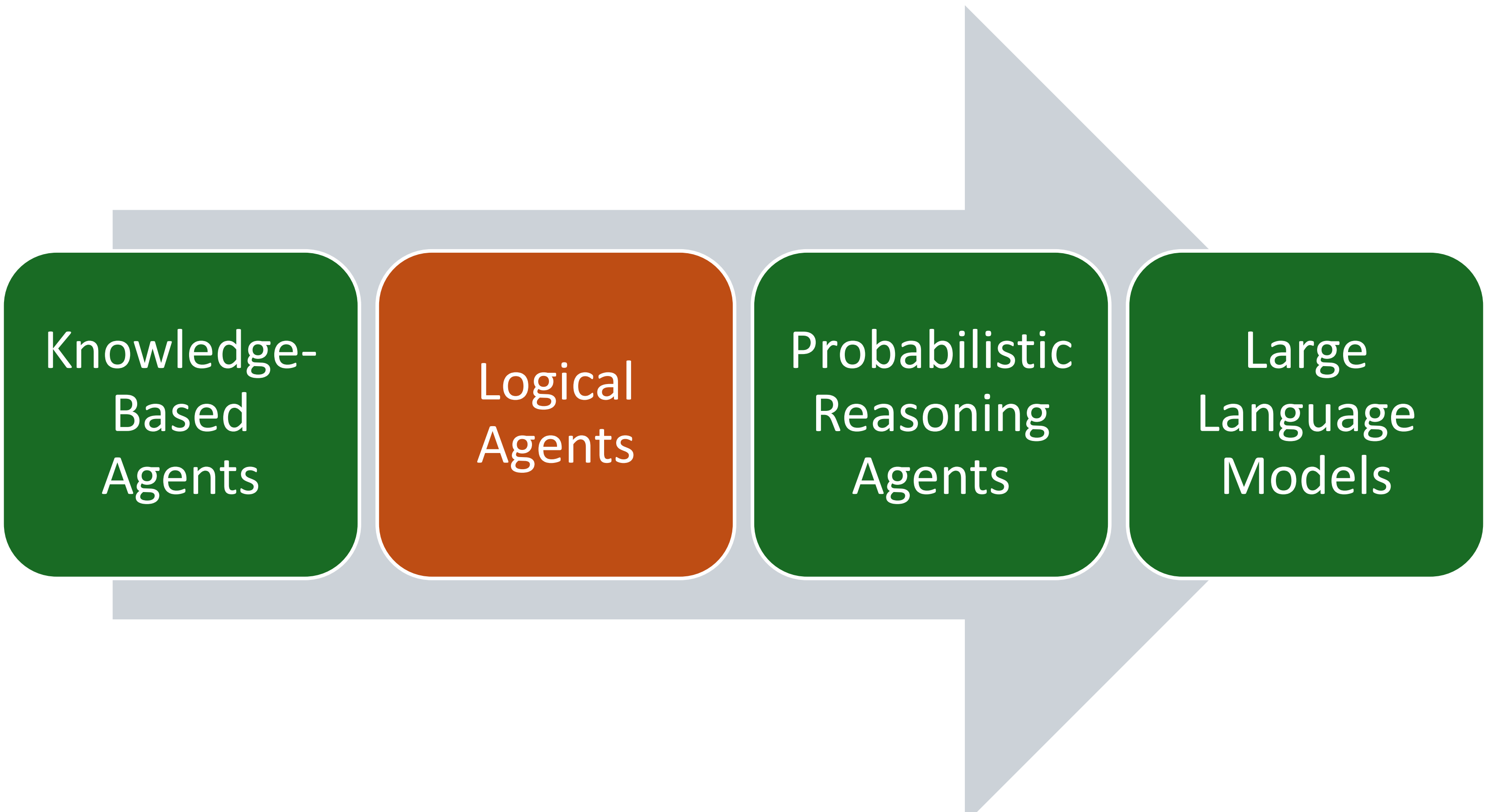
Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

+ Natural Language word patterns representing facts, objects, relations, … ???

Knowledge-based Agents

Logical and Probabilistic Reasoning Agents

Outline


Knowledge-Based Agents
Logical Agents
Probabilistic Reasoning Agents
Large Language Models

Logical Agents

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
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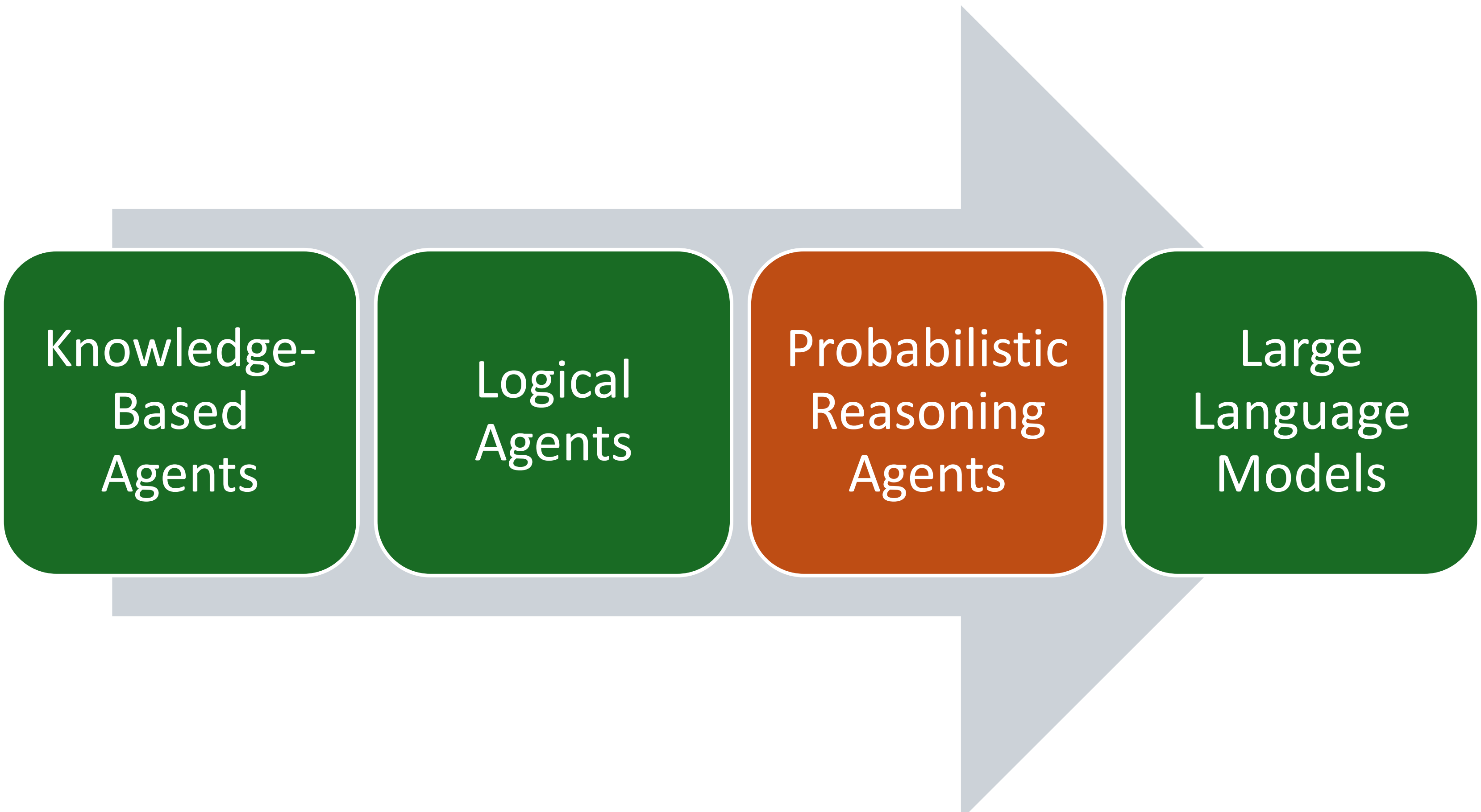
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

- Facts are logical sentences that are known to be true.
- Inference: Generate new sentences that are entailed by all known sentences.
- Implementation: Typically using Prolog
  - Declarative logic programing language.
  - Runs queries over the program (= the knowledge base)
- Synonyms: Symbolic AI, Expert Systems

Issues:

- Inference is computationally very expensive.
- Logic cannot deal with uncertainty.

Outline



Knowledge-Based Agents
Logical Agents
Probabilistic Reasoning Agents
Large Language Models

Probabilistic Reasoning

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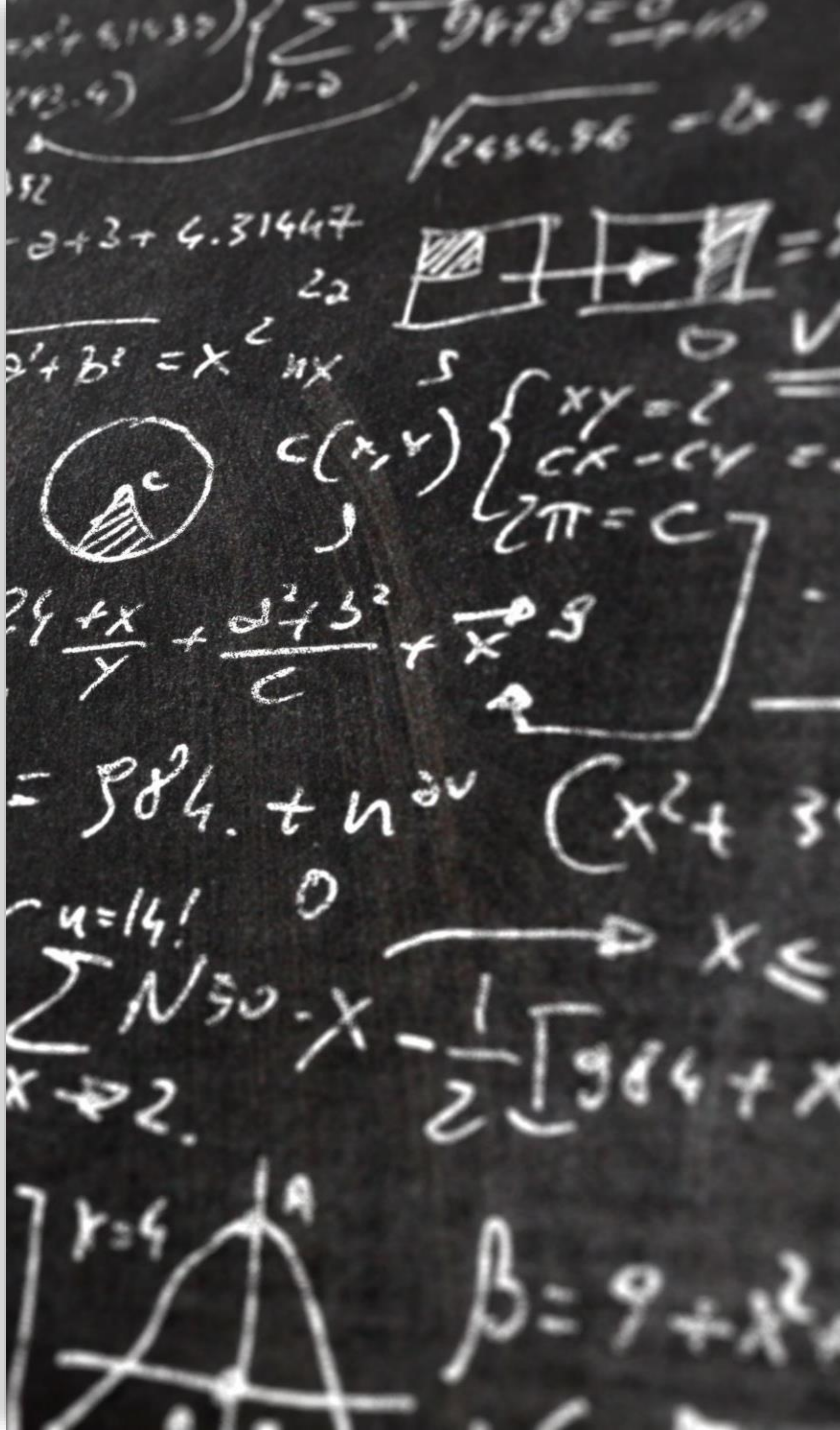
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

- Replaces true/false with a probability.
- This is the basis for
  - Probabilistic reasoning under uncertainty
  - Decision theory
  - Machine Learning

We will talk about these topics a lot more

Conclusion

- The **clear separation between knowledge and inference engine** is very useful.
- **Pure logic** is often not flexible enough. The fullest realization of knowledge-based agents using logic was in the field of expert systems or knowledge-based systems in the 1970s and 1980s.
- **Pretrained Large Language Models** are an interesting new application of knowledge-based agents based on natural language.
- In one of the following chapters, we will talk about **probability theory,** the standard language to reason under uncertainty, and forms the basis of machine learning.



Knowledge-Based Agents using Natural Language

Large Language Models

Outline

- Knowledge-Based Agents
- Logical Agents
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LLMs - Large Language Models

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
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Temporal logic	facts, objects, relations, times	true/false/unknown
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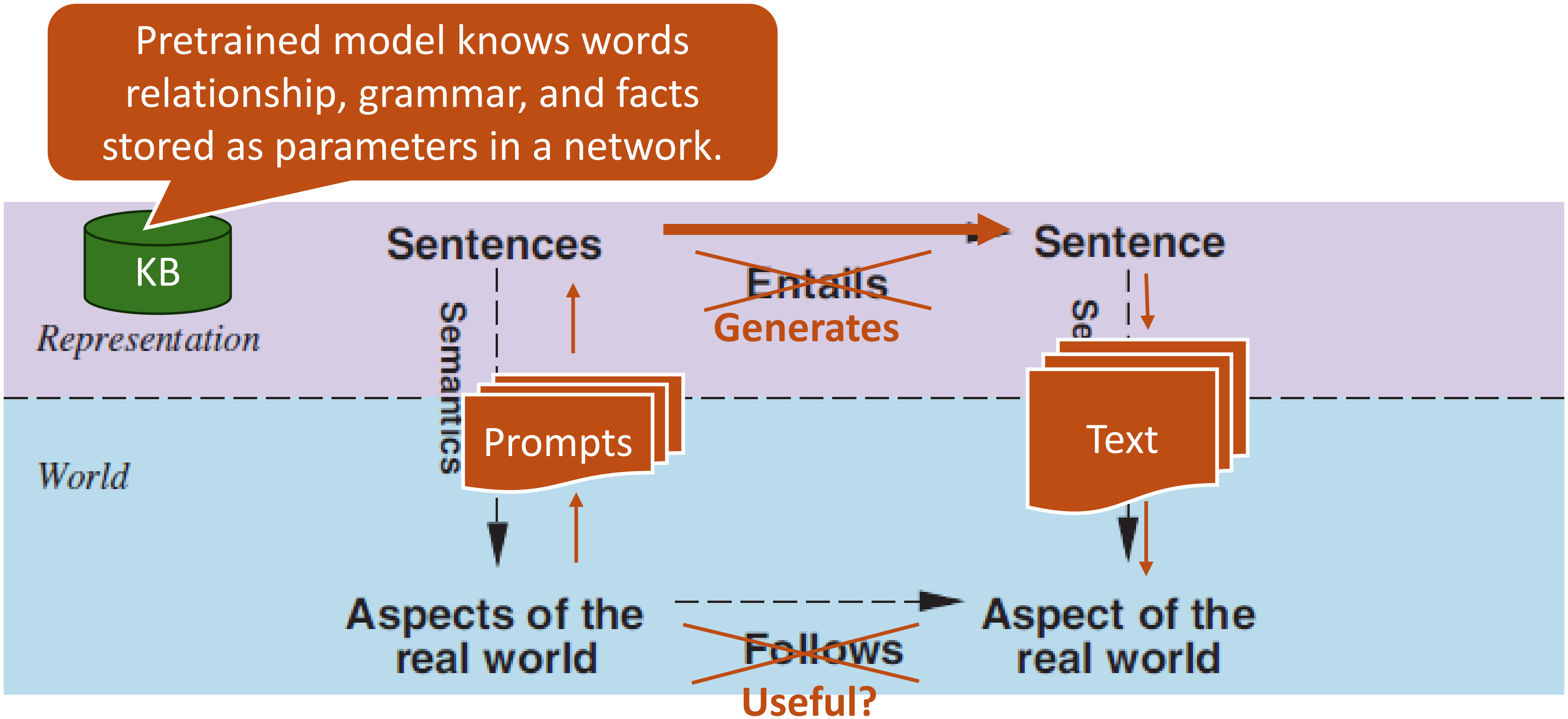
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

\+

Natural Language	word patterns representing facts, objects, relations, …	???

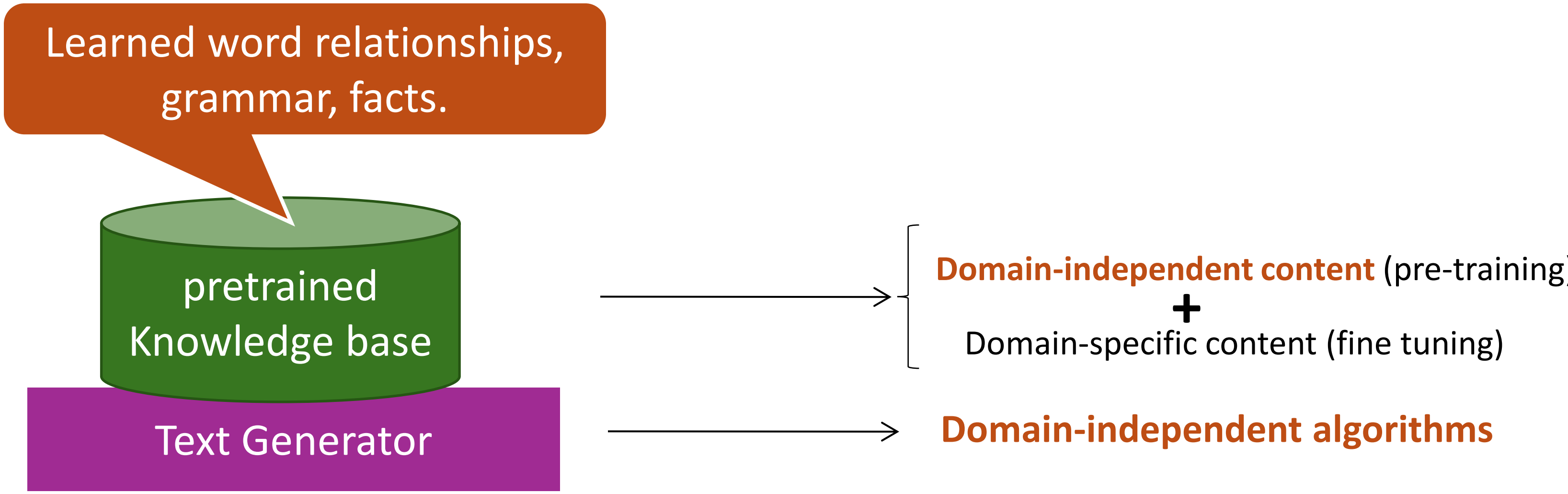
- Extract knowledge from large text corpora.
- Store knowledge compressed as parameters in a deep neural network.

Using Natural Language for Knowledge Representation



- The user formulates a question about the real world as a natural language **prompt** (a sequence of tokens).
- The LLM **generates text** using a pretrained model that represents its knowledge base.
- The text (hopefully) is useful in the real world. The **objective function** is not clear. Maybe it is implied in the prompt?

LLM as a Knowledge-Based Agents



Current text generators are:

- Pretrained decoder-only transformer models (e.g., GPT stands for Generative Pre-trained Transformer). The knowledge base is not updated during interactions.
- Tokens are created autoregressively. One token is generated at a time based on all the previous tokens using the transformer attention mechanism.

LLM as a Generic Knowledge-based Agent

Prompt + already generated tokens

function KB-AGENT(*percept*) **returns** an *action*
  persistent: *KB*, a knowledge base
    t, a counter, initially 0, indicating time

  ~~TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept*, *t*))~~
  action ← ASK(*KB*, MAKE-ACTION-QUERY(*t*))
  ~~TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, *t*))~~
  $t \leftarrow t + 1$
  return *action*

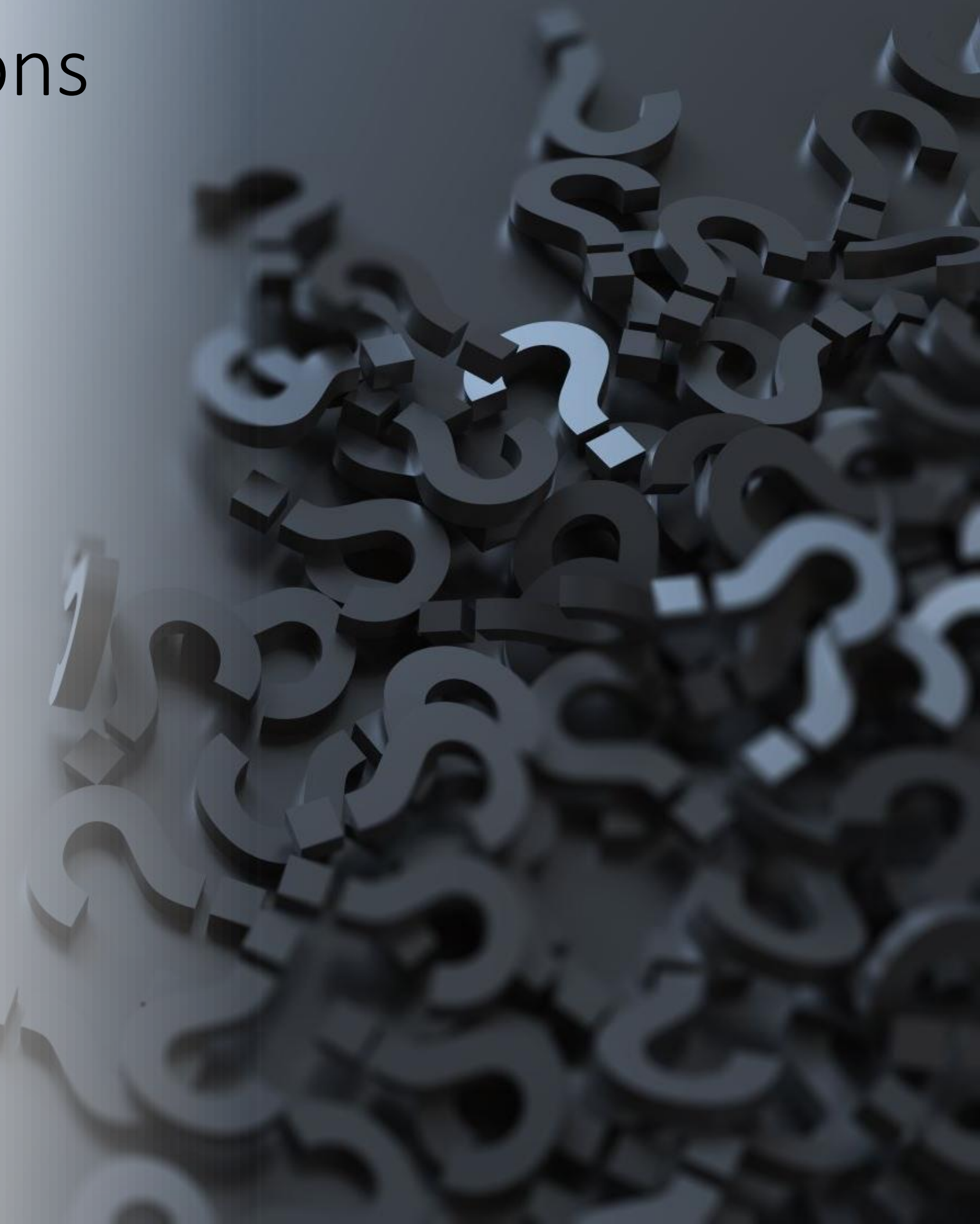
Next token

- A chatbot repeatedly calls the agent function till the agent function returns the ‘end’ token.

Many Open Questions about LLMs

- Correlation is not causation: **Can LLMs reason** to solve problems?
- Leaky data makes it hard to evaluate true **reasoning performance**.
- Generative stochasticity leads to **hallucinations**: LLM makes up facts.
- Autoregression is an exponentially **diverging** diffusion process.
- The training data contains **biases**, nonsense, and harmful content.
- **Security**: LLM can leak sensitive information it was trained on.
- **Rights laundering:** Copyrighted or licensed material can be included in the training data.

Reading: [2307.04821] Amplifying Limitations, Harms and Risks of Large Language Models (arxiv.org)



Appendix: Logic

Details on Propositional and First-Order Logic

Logic to Represent Knowledge

Logic is a formal system for representing and manipulating facts (i.e., knowledge) so that true conclusions may be drawn

Syntax: rules for constructing valid sentences

E.g., $x + 2 \geq y$ is a valid arithmetic sentence, $\geq x2y +$ is not

Semantics: "meaning" of sentences, or relationship between logical sentences and the real world

Specifically, semantics defines truth of sentences

E.g., $x + 2 \geq y$ is true in a world where $x = 5$
and $y = 7$

Propositional Logic

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Propositional Logic: Syntax in Backus-Naur Form

$$
\begin{aligned}
\mathit{Sentence} \quad &\rightarrow \quad \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} \quad &\rightarrow \quad \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots \\
\mathit{ComplexSentence} \quad &\rightarrow \quad (\ \mathit{Sentence}\ ) \\
&\mid \quad \neg\ \mathit{Sentence} \\
&\mid \quad \mathit{Sentence} \land \mathit{Sentence} \\
&\mid \quad \mathit{Sentence} \lor \mathit{Sentence} \\
&\mid \quad \mathit{Sentence} \;\Rightarrow\; \mathit{Sentence} \\
&\mid \quad \mathit{Sentence} \;\Leftrightarrow\; \mathit{Sentence}
\end{aligned}
$$

= Symbols

Negation
Conjunction
Disjunction
Implication
Biconditional

$\textsc{Operator Precedence} \quad : \quad \neg, \land, \lor, \Rightarrow, \Leftrightarrow$

Validity and Satisfiability

A sentence is **valid** if it is true in **all** models/worlds	e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$ are called tautologies and are useful to deduct new sentences.
A sentence is **satisfiable** if it is true in **some** model	e.g., $A \vee B$, C useful to find new facts that satisfy all current possible worlds.
A sentence is **unsatisfiable** if it is true in no models	e.g., $A \wedge \neg A$

Possible Worlds, Models and Truth Tables

A **model** specifies a "possible world" with the true/false status of each proposition symbol in the knowledge base

- E.g., **P** is true and **Q** is true
- With two symbols, there are $2^2 = 4$ possible worlds/models, and they can be enumerated exhaustively using:

A **truth table** specifies the truth value of a composite sentence for each possible assignments of truth values to its atoms. Each row is a model.

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$\boxed{true}$	$true$
$false$	$true$	$true$	$false$	$true$	$\boxed{true}$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$\boxed{true}$	$true$

We have 3 possible worlds for $P \Rightarrow Q = true$

Propositional Logic: Semantics

Rules for evaluating truth with respect to a model:

• $\neg$**P**	is true	iff	**P**	is false			
• **P** $\wedge$ **Q**	is true	iff	**P**	is true	and	**Q**	is true
• **P** $\vee$ **Q**	is true	iff	**P**	is true	or	**Q**	is true
• **P** $\Rightarrow$ **Q**	is true	iff	**P**	is false	or	**Q**	is true

Sentence

Model

Logical Equivalence

Two sentences are logically equivalent iff (read if, and only if) they are true in same models

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Entailment

- **Entailment** means that a sentence follows from the premises contained in the knowledge base:

$$\boldsymbol{KB} \models \boldsymbol{\alpha}$$

- The knowledge base *KB* entails sentence α iff α is true in all models where *KB* is true
  - E.g., KB with x = 0 entails sentence x * y = 0
- Tests for entailment
  - $\boldsymbol{KB} \models \boldsymbol{\alpha}$ iff $(\boldsymbol{KB} \Rightarrow \boldsymbol{\alpha})$ is *valid*
  - $\boldsymbol{KB} \models \boldsymbol{\alpha}$ iff $(\boldsymbol{KB} \land \neg \boldsymbol{\alpha})$ is *unsatisfiable*

Inference

- **Logical inference:** a procedure for generating sentences that follow from (ar entailed by) a knowledge base KB.

- An inference procedure is **sound** if it derives a sentence α iff KB $\models \alpha$. I.e, it only derives **true sentences**.

- An inference procedure is **complete** if it can derive **all** α for which $KB \models \alpha$.

Inference

- How can we check whether a sentence α is entailed by KB?

- How about we **enumerate all possible models of the KB** (truth assignments of all its symbols), and check that α is true in every model in which KB is true?
  - This is sound: All produced answer are correct.
  - This is complete: It will produce all correct answers.
  - **Problem**: if KB contains *n* symbols, the truth table will be of size 2^n

- Better idea: use ***inference rules***, or sound procedures to generate new sentences or *conclusions* given the *premises* in the KB.
- Look at the textbook for inference rules and resolution.

Inference Rules

- Modus Ponens

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$

premises

conclusion

This means: If the KB contains the sentences $\alpha \Rightarrow \beta$ and α then β is true.

- And-elimination

$$\frac{\alpha \wedge \beta}{\alpha}$$

Inference Rules

- And-introduction

$$\frac{\alpha,\ \beta}{\alpha \wedge \beta}$$

- Or-introduction

$$\frac{\alpha}{\alpha \vee \beta}$$

Inference Rules

- Double negative elimination

$$\frac{\neg\neg\alpha}{\alpha}$$

- Unit resolution

$$\frac{\alpha \vee \beta, \neg\beta}{\alpha}$$

Resolution

$$\frac{\alpha \vee \beta, \neg \beta \vee \gamma}{\alpha \vee \gamma} \quad \text{or} \quad \frac{\alpha \vee \beta, \beta \Rightarrow \gamma}{\alpha \vee \gamma}$$

- Example:
  - α: "The weather is dry"
  - β: "The weather is rainy"
  - γ: "I carry an umbrella"

Resolution is Complete

$$\frac{\alpha \vee \beta, \neg \beta \vee \gamma}{\alpha \vee \gamma}$$

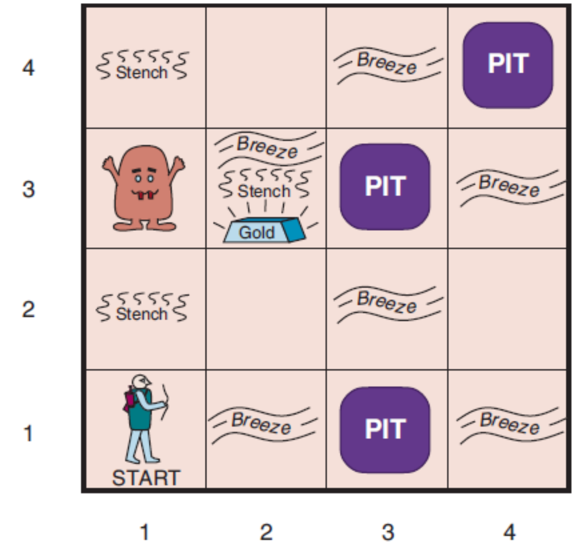
- To prove $KB \models \alpha$, assume $KB \wedge \neg \alpha$ and derive a contradiction
- Rewrite $KB \wedge \neg \alpha$ as a conjunction of *clauses*,
  or disjunctions of *literals*
  - *Conjunctive normal form* (CNF)
- Keep applying resolution to clauses that contain *complementary literals* and adding resulting clauses
  to the list
  - If there are no new clauses to be added, then KB does not entail α
  - If two clauses resolve to form an *empty clause*, we have a contradiction and $KB \models \alpha$

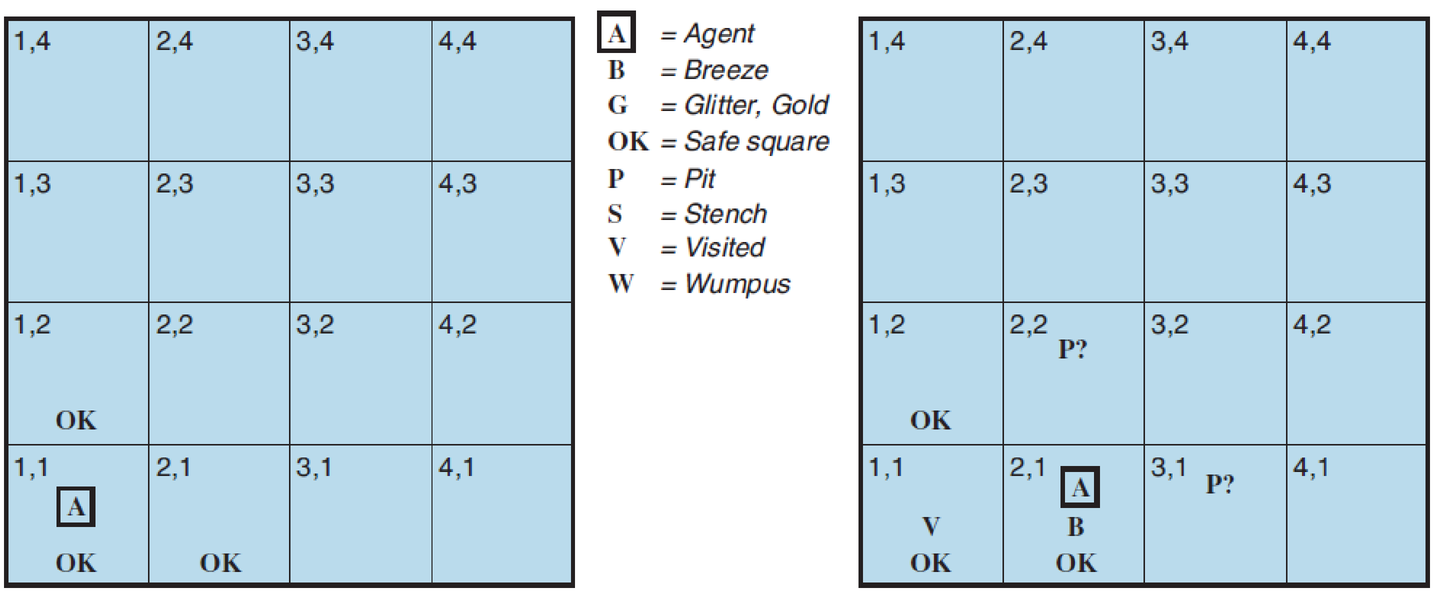
Complexity of Inference

- Propositional inference is ***co-NP-complete***
  - *Complement* of the SAT problem: $\alpha \models \beta$ if and only if the sentence $\alpha \land \neg \beta$ is *unsatisfiable*
  - Every known inference algorithm has worst-case exponential run time complexity.

- Efficient inference is only possible for restricted cases
  - e.g., Horn clauses are disjunctions of literals with at most one positive literal.

Example: Wumpus World





(a) (b)

Example: Wumpus World

Initial KB needs to contain rules like these for each square:

$Breeze(1,1) \Leftrightarrow Pit(1,2) \vee Pit(2,1)$

$Breeze(1,2) \Leftrightarrow Pit(1,1) \vee Pit(1,3) \vee Pit(2,2)$

$\text{Stench}(1,1) \Leftrightarrow W(1,2) \vee W(2,1)$

…

We have to enumerate all possible scenarios in propositional logic! First-order logic can help.

Percepts at (1,1) are no breeze or stench. Add the following facts to the KB:

$\neg Breeze(1,1)$

$\neg Stench(1,1)$

Inference will tell us that the following facts are entailed:

$\neg Pit(1,2), \neg Pit(2,1), \neg W(1,2), \neg W(2,1)$

This means that (1,2) and (2,1) are safe.

Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions.
- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences in models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
- Resolution is complete for propositional logic.
- Algorithms use forward, backward chaining, are linear in time, and complete for special clauses (definite clauses).

Limitations of Propositional Logic

Suppose you want to say “All humans are mortal”

- In propositional logic, you would need ~6.7 billion statements of the form:

  Michael_Is_Human and Michael_Is_Mortal,

  Sarah_Is_Human and Sarah_Is_Mortal, …

Suppose you want to say “Some people can run a marathon”

- You would need a disjunction of ~6.7 billion statements:

  Michael_Can_Run_A_Marathon or … or Sarah_Can_Run_A_Marathon

First-Order Logic

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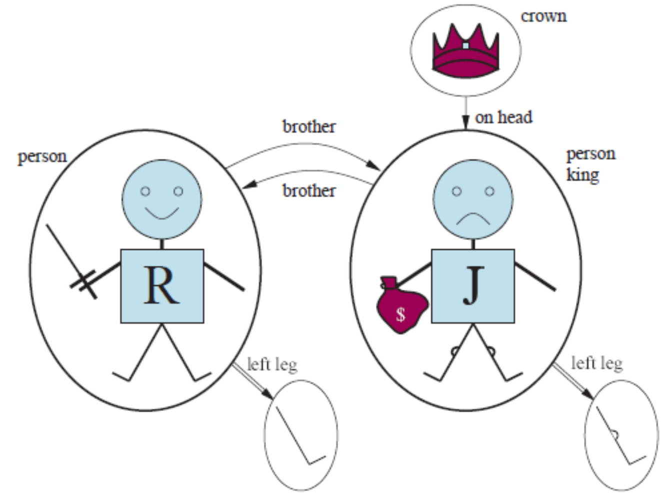
First-order Logic adds **objects** and **relations** to the facts of propositional logic.

This addresses the issues of propositional logic, which needs to store a fact for each instance of and object individually.

Syntax of FOL



$$
\begin{array}{rcl}
\textit{Sentence} & \rightarrow & \textit{AtomicSentence} \mid \textit{ComplexSentence} \\
\textit{AtomicSentence} & \rightarrow & \textit{Predicate} \mid \textit{Predicate}(\textit{Term}, \ldots) \mid \textit{Term} = \textit{Term} \\
\textit{ComplexSentence} & \rightarrow & (\ \textit{Sentence}\ ) \\
& \mid & \neg\ \textit{Sentence} \\
& \mid & \textit{Sentence} \land \textit{Sentence} \\
& \mid & \textit{Sentence} \lor \textit{Sentence} \\
& \mid & \textit{Sentence} \ \Rightarrow \ \textit{Sentence} \\
& \mid & \textit{Sentence} \ \Leftrightarrow \ \textit{Sentence} \\
& \mid & \textit{Quantifier}\ \textit{Variable}, \ldots\ \textit{Sentence} \\
\\
\textit{Term} & \rightarrow & \textit{Function}(\textit{Term}, \ldots) \\
& \mid & \textit{Constant} \\
& \mid & \textit{Variable} \\
\\
\textit{Quantifier} & \rightarrow & \forall \mid \exists \\
\textit{Constant} & \rightarrow & A \mid X_1 \mid \textit{John} \mid \cdots \\
\textit{Variable} & \rightarrow & a \mid x \mid s \mid \cdots \\
\textit{Predicate} & \rightarrow & \textit{True} \mid \textit{False} \mid \textit{After} \mid \textit{Loves} \mid \textit{Raining} \mid \cdots \\
\textit{Function} & \rightarrow & \textit{Mother} \mid \textit{LeftLeg} \mid \cdots \\
\textsc{Operator Precedence} & : & \neg, =, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

Objects

Relations. Predicate is/returns True or False

Function returns an object

Universal Quantification

- $\forall$**x P(x)**

- Example: "Everyone at SMU is smart"
  $\forall$**x At(x,SMU) $\Rightarrow$ Smart(x)**
  Why not $\forall$**x At(x,SMU) $\wedge$ Smart(x)**?

- Roughly speaking, equivalent to the conjunction of all possible instantiations of the variable:
  [At(John, SMU) $\Rightarrow$ Smart(John)] $\wedge$ ...
  [At(Richard, SMU) $\Rightarrow$ Smart(Richard)] $\wedge$ ...

- $\forall$**x P(x)** is true in a model m iff **P(x)** is true with **x** being each possible object in the model

Existential Quantification

- **$\exists x$ P(x)**

- Example: "Someone at SMU is smart"
  $\exists x$ At(x,SMU) $\wedge$ Smart(x)
  Why not **$\exists x$ At(x,SMU) $\Rightarrow$ Smart(x)**?

- Roughly speaking, equivalent to the disjunction of all possible instantiations:
  [At(John,SMU) $\wedge$ Smart(John)] $\vee$
  [At(Richard,SMU) $\wedge$ Smart(Richard)] $\vee$ …

- **$\exists x$ P(x)** is true in a model m iff **P(x)** is true with **x** being some possible object in the model

Properties of Quantifiers

- ∀**x** ∀**y** is the same as ∀**y** ∀**x**
- ∃**x** ∃**y** is the same as ∃**y** ∃**x**
- ∃**x** ∀**y** is not the same as ∀**y** ∃**x**

  ∃**x** ∀**y Loves(x,y)**

  “There is a person who loves everyone”

  ∀**y** ∃**x Loves(x,y)**

  “Everyone is loved by at least one person”

- **Quantifier duality:** each quantifier can be expressed using the other with the help of negation

  ∀**x Likes(x,IceCream)** ¬∃**x**

  ∃**x Likes(x,Broccoli)** ¬∀

Equality

- $\mathbf{Term_1 = Term_2}$ is true under a given model if and only if $\mathbf{Term_1}$ and $\mathbf{Term_2}$ refer to the same object
- E.g., definition of **Sibling** in terms of **Parent**:

  $\forall$**x,y Sibling(x,y)** $\Leftrightarrow$
  [$\neg$(x = y) $\wedge$ $\exists$m,f $\neg$ (m = f) $\wedge$ Parent(m,x) $\wedge$ Parent(f,x) $\wedge$ Parent(m,y) $\wedge$ Parent(f,y)]

Example: The Kinship Domain

- Brothers are siblings

  $\forall$**x,y Brother(x,y)** $\Rightarrow$ **Sibling(x,y)**

- "Sibling" is symmetric

  $\forall$**x,y Sibling(x,y)** $\Leftrightarrow$ **Sibling(y,x)**

- One's mother is one's female parent

  $\forall$**m,c (Mother(c) = m)** $\Leftrightarrow$ **(Female(m)** $\wedge$ **Parent(m,c))**

Example: The Set Domain

- $\forall s\ Set(s) \Leftrightarrow (s = \{\}) \vee (\exists x,s_2\ Set(s_2) \wedge s = \{x|s_2\})$
- $\neg\exists x,s\ \{x|s\} = \{\}$
- $\forall x,s\ x \in s \Leftrightarrow s = \{x|s\}$
- $\forall x,s\ x \in s \Leftrightarrow [\ \exists y,s_2\ (s = \{y|s_2\} \wedge (x = y \vee x \in s_2))]$
- $\forall s_1,s_2\ s_1 \subseteq s_2 \Leftrightarrow (\forall x\ x \in s_1 \Rightarrow x \in s_2)$
- $\forall s_1,s_2\ (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \wedge s_2 \subseteq s_1)$
- $\forall x,s_1,s_2\ x \in (s_1 \cap s_2) \Leftrightarrow (x \in s_1 \wedge x \in s_2)$
- $\forall x,s_1,s_2\ x \in (s_1 \cup s_2) \Leftrightarrow (x \in s_1 \vee x \in s_2)$

Inference in FOL

Inference in FOL is complicated!

1. **Reduction to propositional logic** and then use propositional logic inference.

2. **Directly do inference on FOL (or a subset like definite clauses)**
   - Unification: Combine two sentences into one.
   - Forward Chaining for FOL
   - Backward Chaining for FOL
   - Logical programming (e.g., Prolog)