#### Introduction to Data Mining

Chapter 5 Association Analysis – Basic Concepts and Algorithms

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Based in Slides by Tan, Steinbach, Karpatne, Kumar

# R Code Examples

• Available R Code examples are indicated on slides by the R logo



• The Examples are available at <a href="https://mhahsler.github.io/Introduction\_to\_Data\_Mining\_R\_Examples/">https://mhahsler.github.io/Introduction\_to\_Data\_Mining\_R\_Examples/</a>



#### Topics

- Definition
- Mining Frequent Itemsets (APRIORI)
- Concise Itemset Representation
- Alternative Methods to Find Frequent Itemsets
- Association Rule Generation
- Support Distribution
- Pattern Evaluation

## **Association Rule Mining**

 Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction.

#### **Market-Basket transactions**

#### **TID** Items

- 1 Bread, Milk
- 2 Bread, Diaper, Beer, Eggs
- 3 Milk, Diaper, Beer, Coke
- 4 Bread, Milk, Diaper, Beer
- 5 Bread, Milk, Diaper, Coke

#### Example of Association Rules



 $\{Diaper\} \rightarrow \{Beer\}, \\ \{Milk, Bread\} \rightarrow \{Eggs, Coke\}, \\ \{Beer, Bread\} \rightarrow \{Milk\}, \end{cases}$ 

**Meaning**: A customer who buys Diapers is also very likely to buy Beer.

TID	Bread	Beer	Eggs	
1	1	0	0	
2	1	1	1	
3	0	1	0	
4	0	1	0	
5	1	0	0	

Representation of transactions as a large, sparse 0-1 matrix. Columns (variables) are the items. Sparse means most entries are 0.



## **Definition: Frequent Itemset**

#### Itemset

- A collection of one or more items
  - Example: {Milk, Bread, Diaper}
- k-itemset
  - An itemset that contains k items

#### Support count (σ)

- Frequency of occurrence of an itemset
- E.g.  $\sigma(\{\text{Milk, Bread, Diaper}\}) = 2$

#### Support

- Fraction of transactions that contain an itemset
- E.g.  $s(\{\text{Milk, Bread, Diaper}\})$ =  $\sigma(\{\text{Milk, Bread, Diaper}\}) / |T| = 2/5$
- Support can also be interpreted as a type of correlation between the variables representing the items in the itemset.

#### Frequent Itemset

 An itemset whose support is greater than or equal to a *minsup* threshold.

#### TID Items

- 1 Bread, Milk
- 2 Bread, Diaper, Beer, Eggs
- 3 Milk, Diaper, Beer, Coke
- 4 Bread, Milk, Diaper, Beer
- 5 Bread, Milk, Diaper, Coke

### **Definition: Association Rule**

#### Association Rule

- An expression of the form  $X \rightarrow Y$ , where X and Y are itemsets.
- Typically,
- Example:  $\{Milk, Bread\} \rightarrow \{Diaper\}$

#### Rule Evaluation Metrics

- Support (s): Fraction of transactions that contain both X and Y
- Confidence (c): Measures how often items in Y appear in transactions that contain X

#### TID Items

- 1 Bread, Milk
- 2 Bread, Diaper, Beer, Eggs
- 3 Milk, Diaper, Beer, Coke
- 4 Bread, Milk, Diaper, Beer
- 5 Bread, Milk, Diaper, Coke

#### **Example:**

$$\{Milk, Bread\} \rightarrow \{Diaper\}$$

$$s = \frac{\sigma(\{Milk, Bread, Diaper\})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\{Milk, Bread, Diaper\})}{\sigma(\{Milk, Diaper\})} = \frac{2}{3} = 0.67$$

$$c(X \to Y) = \frac{\sigma(X \cup Y)}{\sigma(X)} = \frac{s(X \cup Y)}{s(X)}$$

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## Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having:
  - support $(X \to Y) \ge minsup$  threshold
  - confidence $(X \rightarrow Y) \geq minconf$  threshold
- Brute-force approach:

  - List all possible association rules.
     Compute the support and confidence for each rule.
     Prune rules that fail the *minsup* and
  - minconf thresholds.

There are too many potential rules! Computationally prohibitive!

## **Mining Association Rules**

#### **TID** Items

1 Bread, Milk

- 2 Bread, Diaper, Beer, Eggs
- 3 Milk, Diaper, Beer, Coke
- 4 Bread, Milk, Diaper, Beer
- 5 Bread, Milk, Diaper, Coke

#### Example of Rules:

 $\{Milk, Diaper\} \rightarrow \{Beer\} (s=0.4, c=0.67) \\ \{Milk, Beer\} \rightarrow \{Diaper\} (s=0.4, c=1.0) \\ \{Diaper, Beer\} \rightarrow \{Milk\} (s=0.4, c=0.67) \\ \{Beer\} \rightarrow \{Milk, Diaper\} (s=0.4, c=0.67) \\ \{Diaper\} \rightarrow \{Milk, Beer\} (s=0.4, c=0.5) \\ \{Milk\} \rightarrow \{Diaper, Beer\} (s=0.4, c=0.5) \\ \}$ 

#### **Observations:**

- All the above rules are binary partitions of the same itemset: {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence.
- Thus, we may decouple the support and confidence requirements.

## **Mining Association Rules**

- Two-step approach:
  - 1. Frequent Itemset Generation
    - Generate all itemsets whose  $support(X) \ge minsup$
  - 2. Rule Generation
    - Generate high-confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset.
- Frequent itemset generation is still computationally expensive.

### **Frequent Itemset Generation**



### **Reducing Number of Candidates**

- The Apriori Principle:
  - If an itemset is frequent, then all of its subsets must also be frequent.
- The apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \ge s(Y)$$

- Support of an itemset can never exceed the support of its subsets.
- This is also known as the anti-monotone property of support.

#### Illustrating the Apriori Principle



**Figure 6.4.** An illustration of support-based pruning. If  $\{a, b\}$  is infrequent, then all supersets of  $\{a, b\}$  are infrequent.

## **Illustrating the Apriori Principle**



Items (1-itemsets)

## Apriori Algorithm

- •Method:
  - Let k = 1
  - Generate frequent itemsets of length 1.
  - Repeat until no new frequent itemsets are identified:
    - Generate length (k + 1) candidate itemsets from length k frequent itemsets.
    - Prune candidate itemsets containing subsets of length k that are infrequent.
    - Count the support of each candidate by scanning all transactions.
    - Eliminate candidates that are infrequent, leaving only those that are frequent.

## Factors Affecting Complexity

- Choice of minimum support threshold
  - Lowering the support threshold results in more frequent itemsets.
  - This may increase number of candidates and max. length of frequent itemsets.
- Dimensionality (number of items) of the data set
  - More **space** is needed to store support count of each item.
  - If the number of frequent items also increases, both computation and I/O costs may also increase.
- Size of database
  - Since the Apriori algorithm makes multiple passes, run time of algorithm may increase with number of transactions.
- Number of items per transaction
  - Transaction width increases with denser data sets.
  - This may increase max. length of frequent itemsets and thus run time.



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## **Maximal Frequent Itemset**

An itemset is maximal frequent if none of its immediate supersets is frequent



Figure 6.16. Maximal frequent itemset.

### **Closed Itemset**

 An itemset is closed if none of its immediate supersets has the same support as the itemset (can only have smaller support -> see APRIORI principle)

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

### Maximal vs. Closed Itemsets



### Maximal vs Closed Frequent Itemsets



### Maximal vs Closed Itemsets



Figure 6.18. Relationships among frequent, maximal frequent, and closed frequent itemsets.



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Alternative Methods for Frequent Itemset Generation

- Traversal of Itemset Lattice
  - Equivalent Classes





(a) Prefix tree.

(b) Suffix tree.

Alternative Methods for Frequent Itemset Generation

•Representation of Database: horizontal vs vertical data layout

Horizontal Data Layout

TID	Items
1	a,b,e
2	b,c,d
3	c,e
4	a,c,d
5	a,b,c,d
6	a,e
7	a,b
8	a,b,c
9	a,c,d
10	b

#### Vertical Data Layout

а	b	С	d	е
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

Figure 6.23. Horizontal and vertical data format.

## **Alternative Algorithms**

- FP-growth
  - Use a compressed representation of the database using an FP-tree
  - Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets
- ECLAT
  - Store transaction id-lists (vertical data layout).
  - Performs fast tid-list intersection (bit-wise XOR) to count itemset frequencies

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### **Rule Generation**

Given a frequent itemset L, find all non-empty subsets  $X = f \subset L$  and Y = L - f such that  $X \rightarrow Y$  satisfies the minimum confidence requirement

$$c(X \to Y) = \frac{\sigma(X \cup Y)}{\sigma(X)}$$

If {A,B,C,D} is a frequent itemset, candidate rules:

ABC $\rightarrow$ D,	$ABD \rightarrow C$ ,	$ACD \rightarrow B$ ,	BCD $\rightarrow$ A,
$A \rightarrow BCD$ ,	$B \rightarrow ACD$ ,	$C \rightarrow ABD$ ,	$D \rightarrow ABC$
$AB \rightarrow CD$ ,	$AC \rightarrow BD$ ,	$AD \rightarrow BC$ ,	$BC \rightarrow AD,$
$BD \rightarrow AC,$	$CD \rightarrow AB$ ,		

If |L| = k, then there are  $2^k - 2$  candidate association rules (ignoring rules  $L \rightarrow \emptyset$  and  $\emptyset \rightarrow L$ ).

## **Rule Generation**

How do we efficiently generate rules from frequent itemsets?

In general, confidence does not have an anti-monotone property

 $c(ABC \rightarrow D)$  can be larger or smaller than  $c(AB \rightarrow D)$ 

- But confidence of rules generated from the same itemset has an anti-monotone property
- e.g., L = {A,B,C,D}:

 $c(ABC \rightarrow D) \ge c(AB \rightarrow CD) \ge c(A \rightarrow BCD)$ 

• Confidence is anti-monotone w.r.t. number of items on the RHS of the rule.

However, most tools only create rules with a single item as the confident:  $X \rightarrow y$ 



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### **Effect of Support Distribution**

Many real data sets have skewed support distribution.

Example: Support distribution of a retail data set



### **Effect of Support Distribution**

- How do we set the appropriate *minsup* threshold?
  - If minsup is set too high, we could miss itemsets involving interesting rare items (e.g., expensive products).
  - If *minsup* is set too low, it is computationally expensive and the number of itemsets is very large.

 Note: Using a single minimum support threshold may not be effective. Algorithms with multiple support thresholds exist.

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### Pattern Evaluation

- Association rule algorithms tend to produce too many rules. Many of them are
  - Uninteresting, or
  - Redundant.
- Interestingness measures can be used to prune/rank the derived patterns.
- A rule {A,B,C} → {D} can be considered redundant if the more general rule {A,B} → {D} has the same or higher confidence.

#### **Application of Interestingness Measure**



## **Computing Interestingness Measure**

Given a rule  $X \rightarrow Y$ , information needed to compute rule interestingness can be obtained from a contingency table (a count table).

Contingency table for  $X \rightarrow Y$ 

	Y	$\overline{Y}$	
X	f <sub>11</sub>	f <sub>10</sub>	f <sub>1+</sub>
$\overline{X}$	f <sub>01</sub>	f <sub>00</sub>	f <sub>o+</sub>
	f <sub>+1</sub>	f <sub>+0</sub>	T

 $\begin{array}{l} f_{11} : \mbox{ support count of X and Y} \\ f_{10} : \mbox{ support count of X and not Y} \\ f_{01} : \mbox{ support count of not X and Y} \\ f_{00} : \mbox{ support count of not X and not Y} \end{array}$ 

error

Used to define various measures e.g., support, confidence, lift, Gini, J-measure, etc.

 $\sup(\{X,Y\}) = \frac{f_{11}}{|T|} \qquad \text{estimates } P(X,Y)$  $conf(X \to Y) = \frac{f_{11}}{f_{1+}} \qquad \text{estimates } P(Y \mid X)$ 

#### Drawback of Confidence

	Coffee	Coffee	
Теа	15	5	20
Tea	75	5	80
	90	10	100

#### Association Rule: Tea $\rightarrow$ Coffee

Support = P(Coffee, Tea) = 15/100 = 0.15

Confidence= P(Coffee | Tea) = 15/20 = **0.75** 

The high confidence is misleading! The measure ignors: P(Coffee) = 90/100 = **0.9** P(Coffee|Tea) = 75/80 = **0.9375** 

### Statistical Independence

Example: A population of 1000 students with

- 600 students know how to swim (S)
- 700 students know how to bike (B)
- 450 students know how to swim and bike (S, B)
- P(S,B) = 450/1000 = 0.45 (observed joint prob.)
- $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$  (expected under indep.)

From probability theory we know:

- $P(S,B) = P(S) \times P(B) =>$  Statistical independence
- $P(S, B) > P(S) \times P(B) =>$  Positively correlated
- $P(S,B) < P(S) \times P(B) =>$  Negatively correlated

#### **Statistical-based Measures**

Measures that take statistical dependence into account for rule:  $X \rightarrow Y$ 

$$Lift = Interest = \frac{P(Y|X)}{P(Y)} = \frac{P(X,Y)}{P(X)P(Y)}$$
  
Deviation from independence  
$$PS = P(X,Y) - P(X)P(Y)$$
  
$$\Phi = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1 - P(X)]P(Y)[1 - P(Y)]}}$$
  
Phi correlation (= correlation between 0-1 vectors)

### Example: Lift/Interest

	Coffee	Coffee	
Теа	15	5	20
Tea	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Conf(Tea → Coffee)= P(Coffee|Tea) = P(Coffee,Tea)/P(Tea) = .15/.2 = 0.75

but P(Coffee) = 0.9

 $\Rightarrow Lift(Tea \Rightarrow Coffee) = P(Coffee,Tee)/(P(Coffee)P(Tee))$ = .15/(.9 x .2) = **0.8333** 

Note: Lift < 1, therefore Coffee and Tea are negatively associated

Many measures have been proposed in the literature

Some measures are good for certain applications, but not for others

What criteria should we use to determine whether a measure is good or bad?

What about Apriori-style support-based pruning? How does it affect these measures?

**Source**: The list is from Pang-Ning Tan, Vipin Kumar, and Jaideep Srivastava. Selecting the right objective measure for association analysis. Information Systems, 29(4):293--313, 2004.

A larger list of measures is available at: <u>A Probabilistic</u> Comparison of Commonly Used Interest Measures for <u>Association Rules</u>

[	#	Measure	Definition
ſ	1	$\phi$ -coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
	2	Goodman-Kruskal's ( $\lambda$ )	$\frac{\sum_{j} \max_{k} P(A_j, B_k) + \sum_{k} \max_{j} P(A_j, B_k) - \max_{j} P(A_j) - \max_{k} P(B_k)}{2 - \max_{j} P(A_j) - \max_{k} P(B_k)}$
	3	Odds ratio $(\alpha)$	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},\overline{B})}$
	4	Yule's $Q$	$\frac{P(A,B)P(\overline{AB}) - P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB}) + P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha - 1}{\alpha + 1}$
	5	Yule's $Y$	$\frac{\sqrt{P(A,B)P(\overline{AB})} - \sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})} + \sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
	6	Kappa ( $\kappa$ )	$\frac{P(A,B)P(\overline{A},\overline{B}) - P(A,B)P(\overline{A},B)}{P(A,B) - P(\overline{A})P(\overline{B})}$ $\frac{P(A,B) + P(\overline{A},\overline{B}) - P(A)P(B) - P(\overline{A})P(\overline{B})}{1 - P(A)P(B) - P(\overline{A})P(\overline{B})}$
	7	Mutual Information $(M)$	$\frac{\sum_{i} \sum_{j} P(A_i, B_j) \log \frac{P(A_i, B_j)}{P(A_i) P(B_j)}}{\min(-\sum_{i} P(A_i) \log P(A_i), -\sum_{j} P(B_j) \log P(B_j))}$
	8	J-Measure $(J)$	$\max\left(P(A,B)\log(\frac{P(B A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B} A)}{P(\overline{B})})\right),$
	9	Gini index $(G)$	$P(A, B) \log(\frac{P(A B)}{P(A)}) + P(\overline{A}B) \log(\frac{P(\overline{A} B)}{P(\overline{A})}))$ $\max\left(P(A)[P(B A)^2 + P(\overline{B} A)^2] + P(\overline{A})[P(B \overline{A})^2 + P(\overline{B} \overline{A})^2] - P(B)^2 - P(\overline{B})^2,$
			$P(B)[P(A B)^{2} + P(\overline{A} B)^{2}] + P(\overline{B})[P(A \overline{B})^{2} + P(\overline{A} \overline{B})^{2}] - P(A)^{2} - P(\overline{A})^{2})$
	10	Support $(s)$	P(A,B)
	11	Confidence $(c)$	$\max(P(B A), P(A B))$
	12	Laplace $(L)$	$\max\left(\frac{NP(A,B)+1}{NP(A)+2},\frac{NP(A,B)+1}{NP(B)+2}\right)$
	13	Conviction $(V)$	$\max\left(\frac{P(A)P(B)}{P(A\overline{B})}, \frac{P(B)P(A)}{P(B\overline{A})}\right)$
	14	Interest $(I)$	$\frac{P(A,B)}{P(A)P(B)}$
	15	$\cosine(IS)$	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
	16	Piatetsky-Shapiro's $(PS)$	P(A,B) - P(A)P(B)
	17	Certainty factor $(F)$	$\max\left(\frac{P(B A) - P(B)}{1 - P(B)}, \frac{P(A B) - P(A)}{1 - P(A)}\right)$
	18	Added Value $(AV)$	$\max(P(B A) - P(B), P(A B) - P(A))$
	19	Collective strength $(S)$	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
	20	Jaccard $(\zeta)$	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
	21	Klosgen $(K)$	$\sqrt{P(A,B)} \max(P(B A) - P(B), P(A B) - P(A))$

### **Comparing Different Measures**

Experiment: 10 examples of contingency tables.

Example	<b>f</b> <sub>11</sub>	<b>f</b> <sub>10</sub>	<b>f</b> <sub>01</sub>	<b>f</b> <sub>00</sub>
E1	8123	83	424	1370
E2	8330	2	622	1046
E3	9481	94	127	298
E4	3954	3080	5	2961
E5	2886	1363	1320	4431
E6	1500	2000	500	6000
E7	4000	2000	1000	3000
E8	4000	2000	2000	2000
E9	1720	7121	5	1154
E10	61	2483	4	7452

Rankings of contingency tables using various measures:

– support & confidence

					-		-	-		-			-							
$\phi$	λ	α	Q	Y	κ	M	J	G	s	с	L	V	Ι	IS	PS	F	AV	S	ሪ	K
1	1	3	3	3	1	2	2	1	3	-5	5	4	6	2	2	4	6	1	2	5
2	2	1	1	1	2	1	3	2	2	1	1	1	8	3	5	1	8	2	3	6
3	3	4	4	4	3	3	8	7	1	4	4	6	10	1	8	6	10	3	1	10
4	7	2	2	2	5	4	1	3	6	2	2	2	4	4	1	2	3	4	5	1
5	4	8	8	8	4	7	5	4	7	9	9	9	3	6	3	9	4	5	6	3
6	6	7	7	7	7	6	4	6	9	8	8	7	2	8	6	7	2	7	8	2
7	5	9	9	9	6	8	6	5	4	7	7	8	5	5	4	8	5	6	4	4
8	9	10	10	10	8	10	10	8	4	10	10	10	9	7	7	10	9	8	7	9
9	9	5	5	5	9	9	7	9	8	3	3	3	7	9	9	3	7	9	9	8
10	8	6	6	6	10	5	9	10	10	6	6	5	1	10	10	5	1	10		
lift														R						
	φ 1 2 3 4 5 6 7 8 9 10	$\begin{array}{c c} \phi & \lambda \\ 1 & 1 \\ 2 & 2 \\ 3 & 3 \\ 4 & 7 \\ 5 & 4 \\ 6 & 6 \\ 7 & 5 \\ 8 & 9 \\ 9 & 9 \\ 9 & 9 \\ 10 & 8 \end{array}$	$\begin{array}{c c c c c c c } \phi & \lambda & \alpha \\ \hline 1 & 1 & 3 \\ 2 & 2 & 1 \\ 3 & 3 & 4 \\ 4 & 7 & 2 \\ 5 & 4 & 8 \\ 6 & 6 & 7 \\ 7 & 5 & 9 \\ 8 & 9 & 10 \\ 9 & 9 & 5 \\ 10 & 8 & 6 \\ \end{array}$	$\begin{array}{c c c c c c c } \phi & \lambda & \alpha & Q \\ \hline 1 & 1 & 3 & 3 \\ 2 & 2 & 1 & 1 \\ 3 & 3 & 4 & 4 \\ 4 & 7 & 2 & 2 \\ 5 & 4 & 8 & 8 \\ 6 & 6 & 7 & 7 \\ 7 & 5 & 9 & 9 \\ 8 & 9 & 10 & 10 \\ 9 & 9 & 5 & 5 \\ 10 & 8 & 6 & 6 \\ \end{array}$	$\begin{array}{c c c c c c c c } \phi & \lambda & \alpha & Q & Y \\ \hline 1 & 1 & 3 & 3 & 3 \\ 2 & 2 & 1 & 1 & 1 \\ 3 & 3 & 4 & 4 & 4 \\ 4 & 7 & 2 & 2 & 2 \\ 5 & 4 & 8 & 8 & 8 \\ 6 & 6 & 7 & 7 & 7 \\ 7 & 5 & 9 & 9 & 9 \\ 8 & 9 & 10 & 10 & 10 \\ 9 & 9 & 5 & 5 & 5 \\ 10 & 8 & 6 & 6 & 6 \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $					$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$						

### **Support-based Pruning**

- Most of the association rule mining algorithms use support measure to prune rules and itemsets.
- Study effect of support pruning on correlation of itemsets:
  - Generate 10,000 random contingency tables.
  - Compute support and pairwise correlation for each table.
  - Apply support-based pruning and examine the tables that are removed.

## The Effect of Support-based Pruning









Support-based pruning eliminates mostly negatively correlated itemsets!

# Subjective Interestingness Measure

- Objective measures
  - Rank patterns based on statistics computed from data.
  - e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).
- Subjective measures
  - Rank patterns according to user's interpretation.
    - A pattern is subjectively interesting if it **contradicts the expectation** of a user (Silberschatz & Tuzhilin)
    - A pattern is subjectively interesting if it is **actionable** (Silberschatz & Tuzhilin)

### Interestingness via Unexpectedness

Need to model the expectation of users (domain knowledge)



 Need to combine the expectation of users with evidence from data (i.e., extracted patterns)

#### Conclusion

Association rule mining has many applications where data can be seen as large transaction data sets.

- Market Basket Analysis
   Marketing & Retail. E.g., frequent itemsets give
   information about "other customer who bought
   this item also bought X"
- Exploratory Data Analysis
   Find correlation in very large (= many transactions), high-dimensional (= many items) data
- Intrusion Detection Rules with low support but very high lift
- Build Rule-based Classifiers Class association rules (CARs)